AP Calculus BC
Unit 9 – Review

Name: Answer Key*

1. Determine whether the sequence $\left\{\frac{\ln n}{n^2}\right\}$ converges. Justify your answer.

Converges to 2

2. For which of the following is the nth Term Test for Divergence appropriate? Explain. (There may be more than one correct answer.)

(a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

diverges by nth term test

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

n^{th} term test is not appropriate

(c) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

diverges by nth term test

(d) $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!}$

nth term test is not appropriate

3. Evaluate the sum: $\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

$\frac{5}{6}$

4. Use the indicated test to determine whether the series converges or diverges. If that test is not appropriate, explain why. If it is possible to find the value to which the series converges, do so.

(a) Geo Series: $3+\frac{15}{4}+\frac{75}{16}+\frac{375}{64}+\cdots$

diverges

(b) Geo Series: $\sum_{n=1}^{\infty} \frac{1}{2^n}$

converges to 1

(c) p-Series: $\sum_{n=1}^{\infty} n^{-2/3}$

diverges

(d) Integral Test: $\sum_{n=1}^{\infty} \frac{5n}{2n^2+3}$

diverges

(e) Direct Comparison: $\sum_{n=1}^{\infty} \frac{e^n}{n+3}$

Direct Comparison not appropriate

(f) Direct Comparison: $\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$

Converges

(g) Limit Comparison: $\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$

diverges

(h) Limit Comparison: $\sum_{n=1}^{\infty} \frac{3^n}{3n(4^n)}$

converges

(i) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

converges

(j) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2-2n}$

NOT appropriate test

(k) Alt Series Test: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

converges

(l) Alt. Series Test: $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{2n}$

Diverges
(by nth term test)

(m) Root Test: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

converges

(n) Root Test: $\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^{2n}$

diverges

5. How many terms are needed to approximate $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ to within 0.001?

4

6. Consider the sequence $\left\{\frac{2n^2}{3n+1}\right\}$.

a. Show the sequence is monotonic for all $n \geq 1$.

monotonic

b. Is the sequence bounded? Justify your answer.

not bounded

c. Does the sequence converge? Give a reason for your answer. Diverges

7. Does the sequence $\left\{\frac{(n+2)!}{(n+3)!}\right\}$ converge or diverge? Explain. If it converges, find its limit.

converges to 0.

8. Find the sum, if it exists: $\sum_{n=0}^{\infty} -3\left(\frac{\sqrt{5}}{2}\right)^n$

diverges

9. Find the sum, if it exists: $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$

9

CONVERGENCE/DIVERGENCE OF SERIES: Determine whether each series converges or diverges. **Show the work and identify the test that you use to confirm your answer.**

10. $\sum_{n=2}^{\infty} \left(\frac{1}{n} - \frac{1}{n-1}\right)$

converges to -1
(Telescoping Series Test

11. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$

converges
(Alternating Series Test)

12. $\sum_{n=1}^{\infty} \left(\frac{6n+1}{4n-3}\right)^{2n}$

diverges
(root test)

13. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

converges
(Root Test)

14. $\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$

diverges
(ratio test)

15. $\sum_{n=1}^{\infty} \frac{2}{\sqrt[3]{n^7}}$

converges
(p-series test)

16. $\sum_{n=1}^{\infty} \frac{e^n}{e^n+1}$

diverges
(integral Test)

17. $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n-3)!}$

diverges
(nth term test)

18. Find the number of terms necessary to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+4}$ with an error less than 0.0001. (Yes, you can figure this out by hand!)

497 terms

19. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges? Explain your answer.

A. $p \geq 2$ B. $p > 2$ C. $p \geq 1$ D. $p > 1$

20. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ is convergent or divergent. If convergent, classify the series as absolutely or conditionally convergent. Show the work that leads to your answer.

absolutely convergent

① $\left\{ \frac{\ln n}{n^2} \right\}$

$$\lim_{n\to\infty}\left(\frac{\ln n}{n^2}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{n}}{2n}\right) = \lim_{n\to\infty}\left(\frac{1}{n}\cdot\frac{2n}{1}\right)$$

$\frac{\infty}{\infty}$

L'Hopital's Rule

$$= \lim_{n\to\infty}\left(\frac{2n}{n}\right)$$

$$= \lim_{n\to\infty} 2 = 2$$

$\boxed{\text{converges to } 2}$

② ⓐ $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$ nth term test

$$\lim_{n\to\infty}\left(\frac{1+3n^2+n^3}{4n^3-5n+2}\right) = \frac{1}{4} \neq 0$$

$\boxed{\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2} \text{ diverges by nth term test}}$

ⓑ $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $\lim_{n\to\infty}\left(\frac{1}{n^2}\right) = \frac{1}{\infty} = 0$

$\boxed{\text{nth term test is not useful}}$

[would use p-series test]

ⓒ $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

$$\lim_{n\to\infty} \left(\frac{n!}{2n!+1}\right) = \frac{1}{2} \neq 0$$

$$\boxed{\sum_{n=1}^{\infty} \frac{n!}{2n!+1} \text{ diverges by } n^{th} \text{ term test}}$$

ⓓ $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!}$

$$\left.\lim_{n\to\infty} \frac{(n+2)!}{10n!}\right\} \text{ you can't rewrite this}$$

$$\boxed{\therefore \text{ nth term test is not appropriate}}$$

③ $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$ Telescoping Series Test

$$= \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \dots + \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+3}\right)$$

$$\lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{n+3}\right) = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \boxed{\frac{5}{6}}$$

Converges

4

(a) Geo series : $3 + \frac{15}{4} + \frac{75}{16} + \frac{375}{64} + \dots$

$\frac{15}{4} \div 3 = \frac{5}{4}$

$$\sum_{n=0}^{\infty} 3\left(\frac{5}{4}\right)^n \qquad r = 5/4$$

$r > 1$, $\boxed{\text{diverges}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ Geo Series

$r < 1$, converges

$S = \frac{a}{1-r}$ you must figure out 1st term since n=0 1st term $\to \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$$= \frac{1/2}{1-\left(\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{1}{2}} = \boxed{1}$$

(c) p-series: $\sum_{n=1}^{\infty} n^{-2/3} = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$p = 2/3$, $p < 1$, $\boxed{\text{diverges}}$

ⓓ Integral Test $\sum_{n=1}^{\infty} \frac{5n}{2n^2+3}$

$\lim_{b\to\infty} \int_1^b \frac{5x}{2x^2+3}\,dx$

$u = 2x^2+3$
$du = 4x\,dx$

$\frac{1}{4}5\int \frac{4\cancel{5}x}{2x^2+3}\,dx$

$\frac{5}{4}\int \frac{du}{u}$

$\frac{5}{4}\ln|2x^2+3|\Big]_1^b$

$\frac{5}{4}\ln|2b^2+3| - \frac{5}{4}\ln|2(1)^2+3|$

$\lim_{b\to\infty}\left(\frac{5}{4}\ln|2b^2+3| - \frac{5}{4}\ln|5|\right)$

$= \infty - \frac{5}{4}\ln 5 = \infty$ $\boxed{\text{diverges}}$

ⓔ $\sum_{n=1}^{\infty} \frac{e^n}{n+3}$ $a_n = \frac{e^n}{n+3}$ $b_n = \frac{e^n}{n}$ bigger

$\sum_{n=1}^{\infty} \frac{e^n}{n} \leftarrow$ Ratio Test
$a_n = \frac{e^n}{n}$ $a_{n+1} = \frac{e^{n+1}}{n+1}$

$\lim_{n\to\infty}\left|\frac{e^{n+1}}{n+1}\cdot\frac{n}{e^n}\right| = \lim_{n\to\infty}\left|\frac{e\cdot n}{n+1}\right| = e > 1$

diverges by Ratio Test

$\therefore$ Direct comparison test is not appropriate b/c if b_n is bigger, then it must converge

(f) Direct Comparison

$$\sum_{n=1}^{\infty} \frac{3^n}{7^n+1} \qquad a_n = \frac{3^n}{7^n+1} \qquad b_n = \frac{3^n}{7^n}$$

bigger

$$\sum_{n=1}^{\infty} \frac{3^n}{7^n} = \sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n$$

geometric series test
$r = 3/7$, $r < 1$
converges

$$\boxed{\sum_{n=1}^{\infty} \frac{3^n}{7^n+1} \text{ converges by Direct Comparison Test}}$$

(g) Limit Comparison

$$\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2} \qquad a_n = \frac{3n+6}{1-5n+7n^2} \qquad b_n = \frac{n}{n^2} = \frac{1}{n}$$

$$\lim_{n\to\infty} \left(\frac{3n+6}{1-5n+7n^2} \cdot \frac{n}{1}\right) = \lim_{n\to\infty} \left(\frac{3n^2+6n}{7n^2-5n+1}\right) = \frac{3}{7} = \checkmark$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \leftarrow \text{p-series test, } p=1\text{, diverges}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2} \text{ diverges by Limit Comparison Test}}$$

(h) Limit Comparison $\sum_{n=1}^{\infty} \frac{3^n}{3n(4^n)}$

$a_n = \frac{3^n}{3n(4^n)}$ $\quad b_n = \frac{3^n}{4^n}$

$\lim_{n\to\infty}\left(\frac{\cancel{3^n}}{3n(\cancel{4^n})} \cdot \frac{\cancel{4^n}}{\cancel{3^n}}\right) = \lim_{n\to\infty} \frac{1}{3n} = \frac{1}{\infty} = 0$ ✓

$\sum_{n=1}^{\infty} \frac{3^n}{4^n} = \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ geometric series test $r<1$, converges

$\boxed{\sum_{n=1}^{\infty} \frac{3^n}{3n(4^n)} \text{ converges by Limit Comparison Test}}$

(i) Ratio Test

$\sum_{n=1}^{\infty} \frac{n^3}{n!}$ $\quad a_n = \frac{n^3}{n!}$ $\quad a_{n+1} = \frac{(n+1)^3}{(n+1)!}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^3}{\underbrace{\cancel{(n+1)!}}_{(n+1)(\cancel{n!})}} \cdot \frac{\cancel{n!}}{n^3}\right|$

$\lim_{n\to\infty}\left(\frac{(n+1)^3}{n^3(n+1)}\right) = \lim_{n\to\infty}\left(\frac{n^3+3n^2+3n+1}{n^4+n^3}\right)$

$= 0$ $\quad \frac{\text{sm degree}}{\text{lg degree}}$

$0 < 1$

$\boxed{\sum_{n=1}^{\infty} \frac{n^3}{n!} \text{ converges by Ratio Test}}$

(j) Ratio Test

$$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2-2n}$$

$a_n = \frac{n^2-1}{n^2-2n}$ $\quad a_{n+1} = \frac{(n+1)^2-1}{(n+1)^2-2(n+1)}$

$= \frac{n^2+2n+1-1}{n^2+2n+1-2n-2}$

$= \frac{n^2+2n}{n^2-1}$

$$\lim_{n\to\infty} \left| \frac{n^2+2n}{\cancel{n^2-1}} \cdot \frac{\cancel{n^2-1}}{n^2-2n} \right|$$

$\boxed{\lim_{n\to\infty} \left| \frac{n^2+2n}{n^2-2n} \right| = 1 \text{, Ratio test is inconclusive}}$

(k) Alt. Series Test

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \qquad a_n = \frac{1}{\sqrt{n}}$$

① $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = \frac{1}{\infty} = 0$ ✓

② $\frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$ ✓

$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \text{ converges by Alt. Series test}}$

(l.) Alt. Series Test

$$\sum_{n=1}^{\infty} \frac{(-1)^n(n+3)}{2n} \qquad a_n = \frac{n+3}{2n}$$

① $$\lim_{n\to\infty}\left(\frac{n+3}{2n}\right) = \frac{1}{2} \neq 0$$

Diverges by nth term test

(m) Root Test

$$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n} \qquad \lim_{n\to\infty} \sqrt[n]{\left|\frac{e^{2n}}{n^n}\right|}$$

$$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\frac{e^2}{n}\right)^{\cancel{n}}}$$

$$\lim_{n\to\infty} \frac{e^2}{n} = \frac{e^2}{\infty} = 0 < 1$$

$$\boxed{\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n} \text{ converges by Root Test}}$$

(n.) Root Test

$$\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^{2n}$$

$$\lim_{n\to\infty} \cancel{\sqrt[n]{\left(\frac{2n}{n+1}\right)^{2n}}}$$

$$\lim_{n\to\infty} \left(\frac{2n}{n+1}\right)^{2} = (2)^2 = 4 > 1$$

$$\boxed{\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^{2n} \text{ diverges by Root Test}}$$

(5) $$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \qquad a_n = \frac{(-1)^n}{(2n)!} \qquad a_{n+1} = \frac{(-1)^{n+1}}{(2(n+1))!} = \frac{(-1)^{n+1}}{(2n+2)!}$$

$$\frac{(-1)^{n+1}}{(2n+2)!} \leq .001$$

$\underline{n=1}$: $\frac{(-1)^{1+1}}{(2(1)+2)!} = \frac{1}{120} = .0083$

$\underline{n=2}$: $\frac{(-1)^{2+1}}{(2(2)+2)!} = \frac{-1}{720} = -.0014$

$\underline{n=3}$: $\frac{(-1)^{3+1}}{(2(3)+2)!} = \frac{1}{40320} = .0000248$

$n=3 \rightarrow$ $\boxed{\text{4 terms from start}}$

(6) $\left\{ \frac{2n^2}{3n+1} \right\}$

(a) monotonic

$\underline{n=1}$ $\frac{2(1)^2}{3(1)+1} = \frac{2}{4} = \frac{1}{2} = 0.50$

$\underline{n=2}$: $\frac{2(2)^2}{3(2)+1} = \frac{8}{7} = 1.14$

$\underline{n=3}$: $\frac{2(3)^2}{3(3)+1} = \frac{18}{10} = 1.8$

$\underline{n=4}$: $\frac{2(4)^2}{3(4)+1} = \frac{32}{13} = 2.46$

increasing → $\boxed{\text{monotonic}}$

(b)

$$\lim_{n\to\infty}\left(\frac{2n^2}{3n+1}\right) = \lim_{n\to\infty}\left(\frac{4n}{3}\right) = \frac{\infty}{3} = \infty$$

$\boxed{\text{not bounded above, bounded below by } 1/2}$

(c) $$\lim_{n\to\infty}\left(\frac{2n^2}{3n+1}\right) = \lim_{n\to\infty}\left(\frac{4n}{3}\right) = \frac{\infty}{3} = \infty$$

$\boxed{\text{diverges}}$

⑦ $\left\{ \frac{(n+2)!}{(n+3)!} \right\}$

$$\lim_{n\to\infty} \left(\frac{\cancel{(n+2)!}}{\cancel{(n+3)!}} \right) = \lim_{n\to\infty} \frac{1}{n+3} = \frac{1}{\infty} = 0$$

$(n+3)(n+2)!$

$\boxed{\text{Converges to } 0}$

⑧ $\sum_{n=0}^{\infty} -3\left(\frac{\sqrt{5}}{2}\right)^n$

Geometric Series Test

$r = \frac{\sqrt{5}}{2} > 1$

$\boxed{\text{diverges}}$

⑨ $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n} = \sum_{n=1}^{\infty} \frac{3^n \cdot 3}{4^n} = \sum_{n=1}^{\infty} 3\left(\frac{3}{4}\right)^n$

Converges by geometric series

1st term: $3\left(\frac{3}{4}\right)^1$

$3 \cdot \frac{3}{4} = \frac{9}{4}$

$$S = \frac{a}{1-r} = \frac{\frac{9}{4}}{1-\frac{3}{4}} = \frac{\frac{9}{4}}{\frac{1}{4}} = \frac{9}{\cancel{4}} \cdot \frac{\cancel{4}}{1} = \boxed{9}$$

(10) $\sum_{n=2}^{\infty} \left(\frac{1}{n} - \frac{1}{n-1} \right)$ Telescoping Series

Start w/ n=2

$$= \left[\left(\cancel{\frac{1}{2}} - \frac{1}{1}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{n}} - \frac{1}{n-1}\right) \right]$$

$$\lim_{n \to \infty} \left(-1 - \frac{1}{n-1} \right) = \boxed{-1}$$

converges by Telescoping Series Test

(11) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$ Alternating Series Test

$$a_n = \frac{1}{2n+3}$$

$$\lim_{n \to \infty} \frac{1}{2n+3} = \frac{1}{\infty} = 0 \checkmark$$

$$\frac{1}{2(n+1)+3} \leq \frac{1}{2n+3} \checkmark$$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$ converges by Alternating Series Test

(12) $\sum_{n=1}^{\infty} \left(\frac{6n+1}{4n-3}\right)^{2n}$ Root Test

$$\lim_{n\to\infty} \cancel{\sqrt[n]{}}\left|\frac{6n+1}{4n-3}\right|^{\cancel{2n}} = \lim_{n\to\infty} \left(\frac{6n+1}{4n-3}\right)^2 = \left(\frac{6}{4}\right)^2$$

$$= \left(\frac{3}{2}\right)^2 = \frac{9}{4} > 1$$

$$\boxed{\sum_{n=1}^{\infty} \left(\frac{6n+1}{4n-3}\right)^{2n} \text{ diverges by root test}}$$

(13) $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{n}$ Root Test

$$\lim_{n\to\infty} \cancel{\sqrt[n]{}}\left|\left(\frac{e}{\pi}\right)^{\cancel{n}}\right| = \lim_{n\to\infty} \left(\frac{e}{\pi}\right) = \frac{e}{\pi} < 1$$

$$\boxed{\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{n} \text{ Converges by Root Test}}$$

(14) $\sum_{n=1}^{\infty} \frac{(n+1)!}{3n}$ Ratio Test

$a_n = \frac{(n+1)!}{3n}$ $a_{n+1} = \frac{(n+1+1)!}{3^{n+1}}$

$$\lim_{n\to\infty} \left| \frac{\overset{(n+2)\cancel{(n+1)!}}{\cancel{(n+2)!}}}{3^{\cancel{n}+1}} \cdot \frac{\cancel{3^n}}{\cancel{(n+1)!}} \right| = \lim_{n\to\infty} \left(\frac{n+2}{3}\right) = \frac{\infty}{3}$$

$= \infty > 1$

$$\boxed{\sum_{n=1}^{\infty} \frac{(n+1)!}{3n} \text{ diverges by ratio test}}$$

(15) $\sum_{n=1}^{\infty} \frac{2}{\sqrt[3]{n^7}}$ p-series test

$\sum_{n=1}^{\infty} \frac{2}{n^{7/3}}$ $p = 7/3$, $p > 1$

converges

$$\boxed{\sum_{n=1}^{\infty} \frac{2}{\sqrt[3]{n^7}} \text{ Converges by p-series test}}$$

(16) $\sum_{n=1}^{\infty} \frac{e^n}{e^n+1}$ Integral Test

$\lim_{b\to\infty} \int_1^b \frac{e^x}{e^x+1}\,dx$ $u = e^x+1$
$du = e^x\,dx$

$\int \frac{du}{u}$

$\ln|e^x+1|\Big]_1^b$

$\lim_{b\to\infty}\left(\ln|e^b+1| - \ln|e^1+1|\right)$

$= \infty - \ln|e+1| = \infty$ diverges

$\boxed{\sum_{n=1}^{\infty} \frac{e^n}{e^n+1} \text{ diverges by Integral Test}}$

(17) $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n-3)!}$ Ratio Test

$a_n = \frac{(n-1)!}{(n-3)!}$ $a_{n+1} = \frac{(n+1-1)!}{(n+1-3)!} = \frac{(n!)}{(n-2)!}$

$\lim_{n\to\infty}\left|\frac{\overset{n\cancel{(n-1)!}}{\cancel{n!}}}{\underset{(n-2)\cancel{(n-3)!}}{\cancel{(n-2)!}}} \cdot \frac{\cancel{(n-3)}!}{\cancel{(n-1)}!}\right| = \lim_{n\to\infty}\left|\frac{n}{n-2}\right| = 1$ Ratio Test is inconclusive

n^{th} term test: $\lim_{n\to\infty} \frac{(n-1)!}{(n-3)!} = \lim_{n\to\infty} \frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$ ↓

17 cont'd $\lim_{n\to\infty} (n-1)(n-2) = \infty \neq 0$

$$\boxed{\sum_{n=1}^{\infty} \frac{(n-1)!}{(n-3)!} \text{ diverges by nth term test}}$$

18 $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+4}$ $a_n = \frac{1}{2n+4}$ $a_{n+1} = \frac{1}{2(n+1)+4} = \frac{1}{2n+6}$

$\frac{1}{2n+6} \leq 0.0001 = \frac{1}{1000}$

$\underline{n=1}$: $\frac{1}{2(1)+6} = \frac{1}{8}$

$\underline{n=2}$ $\frac{1}{2(2)+6} = \frac{1}{10}$

$\underline{n=3}$ $\frac{1}{2(3)+6} = \frac{1}{12}$

$\underline{n=4}$ $\frac{1}{2(4)+6} = \frac{1}{14}$

$\underline{n=5}$ $\frac{1}{2(5)+6} = \frac{1}{16}$

$\frac{1}{2(n)+6} = \frac{1}{1000}$

$2n+6 = 1000$
$2n = 994$
$n = 497$

$\underline{n=497}$: $\frac{1}{2(497)+6} = \frac{1}{1000}$

$\boxed{\text{497 terms}}$

(19) $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges?

$a_n = \frac{n}{n^p+1}$ $b_n = \frac{n}{n^p}$

$a_n = \frac{n}{n^1+1}$ $b_n = \cancel{\frac{n}{n} = 1}$

$a_n = \frac{n}{n^2+1}$ $b_n = \frac{n}{n^2} = \frac{1}{n}$ diverge by p-series

$a_n = \frac{n}{n^3+1}$ $b_n = \frac{n}{n^3} = \frac{1}{n^2}$ converges by p-series

$\boxed{p>2}$

(20) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ $a_n = \frac{1}{n\sqrt{n}} = \frac{1}{n^{3/2}}$

(1) Alt. series test

$\lim_{n\to\infty} \frac{1}{n^{3/2}} = \frac{1}{\infty} = 0$ ✓

$\frac{1}{(n+1)^{3/2}} \leq \frac{1}{n^{3/2}}$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ converges by Alt. series test

(2) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ p-series test $p = 3/2$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series test

$\boxed{\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}} \text{ is absolutely convergent}}$