AP Calculus BC **Name: ______________________________**
Unit 9 – Review

1. Determine whether the sequence $\left\{\dfrac{\ln n}{n^2}\right\}$ converges. Justify your answer.

2. For which of the following is the nth Term Test for Divergence appropriate? Explain. (There may be more than one correct answer.)

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{1+3n^2+n^3}{4n^3-5n+2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{2n!+1}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{(n+2)!}{10n!}$

3. Evaluate the sum: $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$

4. Use the indicated test to determine whether the series converges or diverges. If that test is not appropriate, explain why. If it is possible to find the value to which the series converges, do so.

   (a) Geo Series: $3+\dfrac{15}{4}+\dfrac{75}{16}+\dfrac{375}{64}+\cdots$

   (b) Geo Series: $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^n}$

   (c) p-Series: $\displaystyle\sum_{n=1}^{\infty}n^{-2/3}$

   (d) Integral Test: $\displaystyle\sum_{n=1}^{\infty}\frac{5n}{2n^2+3}$

   (e) Direct Comparison: $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{n+3}$

   (f) Direct Comparison: $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{7^n+1}$

(g) Limit Comparison: $\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$

(h) Limit Comparison: $\sum_{n=1}^{\infty} \frac{3^n}{3n(4^n)}$

(i) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

(j) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2-2n}$

(k) Alt Series Test: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

(l) Alt. Series Test: $\sum_{n=1}^{\infty} \frac{(-1)^n(n+3)}{2n}$

(m) Root Test: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

(n) Root Test: $\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^{2n}$

5. How many terms are needed to approximate $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ to within 0.001?

6. Consider the sequence $\left\{\frac{2n^2}{3n+1}\right\}$.

a. Show the sequence is monotonic for all $n \geq 1$.

b. Is the sequence bounded? Justify your answer.

c. Does the sequence converge? Give a reason for your answer.

7. Does the sequence $\left\{\dfrac{(n+2)!}{(n+3)!}\right\}$ converge or diverge? Explain. If it converges, find its limit.

8. Find the sum, if it exists: $\displaystyle\sum_{n=0}^{\infty} -3\left(\frac{\sqrt{5}}{2}\right)^n$

9. Find the sum, if it exists: $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$

<u>CONVERGENCE/DIVERGENCE OF SERIES:</u> Determine whether each series converges or diverges. **Show the work and identify the test that you use to confirm your answer.**

10. $\displaystyle\sum_{n=2}^{\infty}\left(\frac{1}{n}-\frac{1}{n-1}\right)$

11. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+3}$

12. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{6n+1}{4n-3}\right)^{2n}$

13. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^n$

14. $\displaystyle\sum_{n=1}^{\infty}\frac{(n+1)!}{3^n}$

15. $\displaystyle\sum_{n=1}^{\infty}\frac{2}{\sqrt[3]{n^7}}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{e^n + 1}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{(n-3)!}$

18. Find the number of terms necessary to approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+4}$ with an error less than 0.0001. (Yes, you can figure this out by hand!)

19. What are all values of p for which the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges? Explain your answer.

A. $p \geq 2$

B. $p > 2$

C. $p \geq 1$

D. $p > 1$

20. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ is convergent or divergent. If convergent, classify the series as absolutely or conditionally convergent. Show the work that leads to your answer.