AP Calculus BC **Name:** Answer Key*

Unit 9 – (Days 1 - 4) – QUIZ REVIEW

Test each of the following for convergence or divergence, using each of the following test once. Identify the test you used.

a) **nth-Term Test** ✓
b) **Geometric Series Test** ✓
c) **p-Series Test** ✓
d) **Telescoping Series Test** ✓
e) **Integral Test** ✓
f) **Direct Comparison Test** ✓
g) **Limit Comparison Test** ✓

1) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$ $\frac{n^{1/2}}{n}$ $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ $\boxed{\text{p-series test}}$ $p = 1/2$ since $p<1$, $\boxed{\text{diverges}}$	2) $\sum_{n=0}^{\infty} 5\left(-\frac{1}{5}\right)^n$ $\boxed{\text{Geometric Series Test}}$ $r = -1/5$ since $\lvert r\rvert < 1$, then $\boxed{\text{converges}}$
3) $\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$ $a_n = \frac{1}{3^n+2}$ $b_n = \frac{1}{3^n}$ bigger $\sum_{n=1}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ Geometric Series $r = 1/3$, converges since $r<1$ $\boxed{\therefore \sum_{n=1}^{\infty} \frac{1}{3^n+2} \text{ converges by Direct Comparison Test}}$	4) $\sum_{n=4}^{\infty} \frac{1}{3n^2 - 2n - 15}$ $a_n = \frac{1}{3n^2-2n-15}$ $b_n = \frac{1}{n^2}$ $\lim_{n\to\infty}\left(\frac{1}{3n^2-2n-15} \cdot \frac{n^2}{1}\right)$ $= \lim_{n\to\infty}\left(\frac{n^2}{3n^2-2n-15}\right) = \frac{1}{3} = \#$ ✓ ↓ $\sum_{n=4}^{\infty} \frac{1}{n^2}$ $p = 2$ (p-series test) Since $p>1$, converges ↓ $\boxed{\therefore \sum_{n=4}^{\infty} \frac{1}{3n^2-2n-15} \text{ converges by Limit Comparison Test}}$

5)

$$\sum_{n=1}^{\infty} \frac{n}{2n+3}$$

$$\lim_{n\to\infty}\left(\frac{n}{2n+3}\right) = \frac{1}{2} \neq 0$$

$\boxed{\text{nth-Term test}}$

$\boxed{\text{diverges}}$

6)

$$\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$$

$$\lim_{b\to\infty}\int_1^b \frac{x}{(x^2+1)^2} \qquad u = x^2+1 \quad du = 2x\,dx$$

$$\frac{1}{2}\int \frac{du}{u^2} = \frac{1}{2}\int u^{-2}\,du$$

$$\frac{1}{2}(-1u^{-1})$$

$$\left.\frac{-1}{2(x^2+1)}\right]_1^b$$

$$\frac{-1}{2(b^2+1)} + \frac{1}{2(1^2+1)}$$

$$\lim_{b\to\infty}\left(\frac{-1}{2(b^2+1)} + \frac{1}{4}\right) = \frac{-1}{\infty} + \frac{1}{4} = \frac{1}{4}$$

$\int_1^{\infty} \frac{x}{(x^2+1)^2}$ conv.

$\boxed{\text{By Integral Test}}$

$\boxed{\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2} \text{ converges}}$

7)

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

$$\frac{3}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}$$

$$3 = A(n+3) + B(n)$$

$\underline{n=-3}$: $3 = B(-3)$

$B = -1$

$\underline{n=0}$: $3 = A(0+3)$

$A = 1$

$$3\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+3}$$

$$3\left[\left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{8}}\right) + \ldots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+3}\right)\right]$$

$$\lim_{n\to\infty} 3\left[1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+3}\right]$$

$$= 3\left[1 + \frac{1}{2} + \frac{1}{3}\right] = \frac{11}{2}$$

$\boxed{\text{converges}}$

$\boxed{\text{Telescoping Series Test}}$