AP Calculus BC
Unit 9 – Sequences & Series (Part 1)

Day 6 Notes: Ratio & Root Tests

The RATIO TEST and the ROOT TEST are tests for absolute convergence.

The RATIO TEST works well for series with exponentials and factorials.

<u>RATIO TEST</u>

Let $\sum_{n=1}^{\infty} a_n$ be a series with nonzero terms.

1. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

2. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

3. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the Ratio Test is inconclusive.

Examples: Use the Ratio Test to determine convergence or divergence of each series.

1. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

2. $\sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!}{2^{4n}}$

3. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

The Root Test works great for series with *n*th powers.

ROOT TEST

1. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

2. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

3. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then the Root Test is inconclusive.

Examples: Determine the convergence or divergence of each series with the Root Test.

1. $\sum_{n=1}^{\infty} \left(\frac{\pi}{4}\right)^n$

2. $\sum_{n=1}^{\infty} \left(\frac{-3n}{2n+1}\right)^{3n}$