AP Calculus BC
Unit 9 – Sequences & Series (Part 1)

Day 5 Notes: Alternating Series

An alternating series has terms that alternate between positive and negative:

$$\sum_{n=1}^{\infty}(-1)^n a_n \quad \text{or} \quad \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

For example, this is a common alternating series:

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^n}{n} + \cdots$$

<u>ALTERNATING SERIES TEST</u>

Let $a_n > 0$. The alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converge if both of these conditions are met:

1. $\lim_{n\to\infty} a_n = 0$

2. $a_{n+1} \le a_n$ for all n (each term must be $\le$ the preceding term).

Examples: Determine convergence or divergence.

1. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

2. $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^2+1}$

REMAINDER THEOREM FOR ALTERNATING SERIES

If a convergent alternating series has R_N as the remainder obtained by approximating the sum of the series S with S_N, then

$$|R_N| \leq a_{n+1}$$

*****What this really means**: The remainder after the nth partial sum S_N is always less than or equal to the first omitted term of the alternating series.

Examples:

1. Find the number of terms needed to approximate $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ with an error less than 0.001.

   Start with $|R_N| \leq a_{n+1} \leq 0.001$.

2. Find the number of terms needed to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ with an error less than 0.001.

ABSOLUTE CONVERGENCE OF AN ALTERNATING SERIES

Let $\sum_{n=1}^{\infty} (-1)^n a_n$ be an alternating series.

1. $\sum_{n=1}^{\infty} (-1)^n a_n$ is **absolutely convergent** if $\sum_{n=1}^{\infty} a_n$ converges.

2. $\sum_{n=1}^{\infty} (-1)^n a_n$ is **conditionally convergent** if $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

   but $\sum_{n=1}^{\infty} a_n$ diverges.

Examples: Does each series converge or diverge? If it converges, is it absolutely or conditionally convergent?

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

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Unit 9 – Day 5 – Assignment

#'s 1 – 7: Determine the convergence or divergence of the series.

1) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$	**2)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$
3) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$	**4)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{ln(n+1)}$
5) $\displaystyle\sum_{n=1}^{\infty} sin\left(\frac{(2n-1)\pi}{2}\right)$	**6)** $\displaystyle\sum_{n=1}^{\infty} cos(n\pi)$
7) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$	

#'s 8 – 9: Determine the number of terms required to approximate the sum of the convergent series with an error or less than 0.001.

8) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$	**9)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

#'s 10 – 12: Determine whether the series converges conditionally, or absolutely, or diverges.

10) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$	**11)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$	**12)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{ln n}$