AP Calculus BC Unit 9 – Sequences & Series (Part 1)

Day 5 Notes: Alternating Series

An alternating series has terms that alternate between positive and negative:

$$\sum_{n=1}^{\infty} (-1)^n a_n \text{ or } \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

For example, this is a common alternating series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{(-1)^n}{n} + \dots$$

ALTERNATING SERIES TEST

Let $a_n > 0$. The alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converge if both of these conditions are met: 1. $\lim_{n \to \infty} a_n = 0$ 2. $a_{n+1} \le a_n$ for all n (each term must be \le the preceding term).

Examples: Determine convergence or divergence.

1.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$
 2. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2 + 1}$

REMAINDER THEOREM FOR ALTERNATING SERIES

If a convergent alternating series has R_N as the remainder obtained by approximating the sum of the series S with S_N , then

$$|R_N| \leq a_{n+1}$$

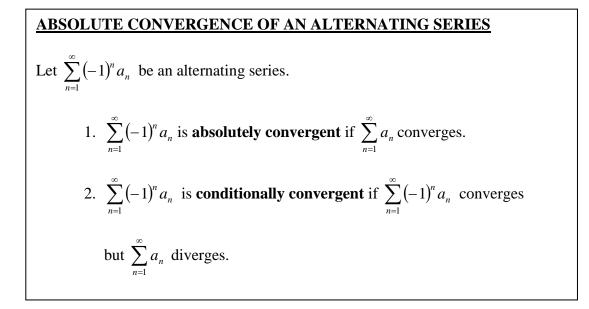
*****What this really means**: The remainder after the *n*th partial sum S_N is always less than or equal to the first omitted term of the alternating series.

Examples:

1. Find the number of terms needed to approximate $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$ with an error less than 0.001.

Start with $|R_N| \le a_{n+1} \le 0.001$.

2. Find the number of terms needed to approximate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ with an error less than 0.001.



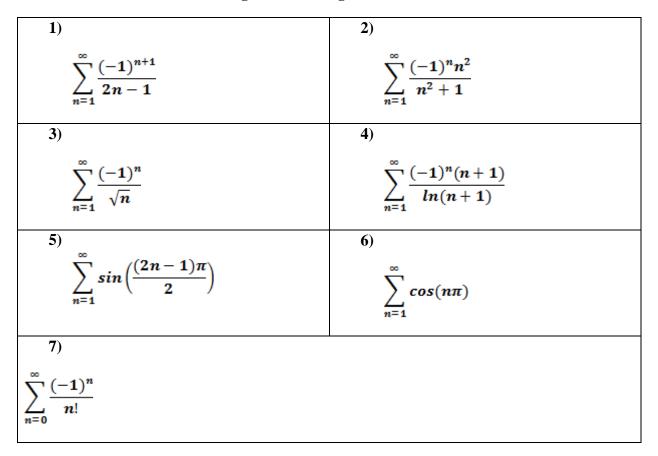
Examples: Does each series converge or diverge? If it converges, is it absolutely or conditionally convergent?

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

$$2. \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$$

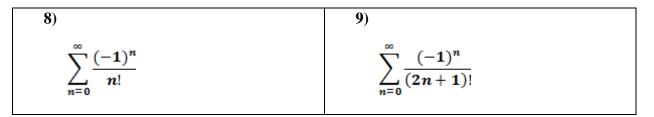
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#'s 1 - 7: Determine the convergence or divergence of the series.

#'s 8-9: Determine the number of terms required to approximate the sum of the convergent series with an error or less than 0.001.



#'s 10 – 12: Determine whether the series converges conditionally, or absolutely, or diverges.

