AP Calculus BC **Name:** ______________________________

Unit 9 – Day 5 – Assignment

#'s 1 – 7: Determine the convergence or divergence of the series.

1) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$	2) $$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$$
3) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$	4) $$\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{ln(n+1)}$$
5) $$\sum_{n=1}^{\infty} sin\left(\frac{(2n-1)\pi}{2}\right)$$	6) $$\sum_{n=1}^{\infty} cos(n\pi)$$
7) $$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$	

#'s 8 – 9: Determine the number of terms required to approximate the sum of the convergent series with an error or less than 0.001.

8) $$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$	9) $$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

#'s 10 – 12: Determine whether the series converges conditionally, or absolutely, or diverges.

10) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$$	11) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$	12) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{lnn}$$

(1) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$

$$a_n = \frac{1}{2n-1}$$

$$\lim_{n\to\infty} \left(\frac{1}{2n-1}\right) = \frac{1}{\infty} = 0 \quad ✓$$

$$\frac{1}{2(n+1)-1} \qquad \frac{1}{2n-1}$$

$$\downarrow$$

$$\frac{1}{2n+1} \leq \frac{1}{2n-1} \quad ✓$$

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ converges by Alternating Series Test

(2) $$\sum_{n=1}^{\infty} \frac{(-1)^{n} n^2}{n^2+1}$$

$$a_n = \frac{n^2}{n^2+1}$$

$$\lim_{n\to\infty} \left(\frac{n^2}{n^2+1}\right) = 1 \neq 0$$

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n} n^2}{n^2+1}$ diverges by n^{th} term test

③ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ $a_n = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$

$\lim_{n \to \infty} \left(\frac{1}{n^{1/2}} \right) = \frac{1}{\infty} = 0$ ✓

$\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$ ✓

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by Alt. Series Test

④ $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{\ln(n+1)}$ $a_n = \frac{(n+1)}{\ln(n+1)}$

$\lim_{n \to \infty} \frac{n+1}{\ln(n+1)} \overset{\frac{\infty}{\infty}}{=} \lim_{n \to \infty} \frac{1}{\frac{1}{n+1}} = \lim_{n \to \infty} (n+1) = \infty \neq 0$

L'Hopital's Rule

$\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{\ln(n+1)}$ diverges by nth term test

(5) $\sum_{n=1}^{\infty} \sin\left(\frac{(2n-1)\pi}{2}\right)$

$n=1$: $\sin\left(\frac{(2(1)-1)\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$n=2$: $\sin\left(\frac{(2(2)-1)\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

$n=3$: $\sin\left(\frac{(2(3)-1)\pi}{2}\right) = \sin\left(\frac{5\pi}{2}\right) = 1$

$n=4$: $= -1$

changes signs $\sum_{n=1}^{\infty} (-1)^{n+1}$

$a_n = 1$

$\lim_{n\to\infty} 1 = 1 \neq 0$

$\sum_{n=1}^{\infty} \sin\left(\frac{(2n-1)\pi}{2}\right) = \sum_{n=1}^{\infty} (-1)^{n+1}$ diverges by nth term test

(6) $\sum_{n=1}^{\infty} \cos(n\pi)$

$n=1$: $\cos(\pi) = -1$
$n=2$: $\cos(2\pi) = 1$
$n=3$: $\cos(3\pi) = -1$
$n=4$: $\cos(4\pi) = 1$

$\Bigg\} \sum_{n=1}^{\infty} (-1)^n$

$a_n = 1$

$\lim_{n\to\infty} 1 = 1 \neq 0$

$\sum_{n=1}^{\infty} \cos(n\pi) = \sum_{n=1}^{\infty} (-1)^n$ diverges by nth term test

(7) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ $\qquad a_n = \frac{1}{n!}$

$\lim_{n\to\infty} \frac{1}{n!} = \frac{1}{\infty} = 0$ ✓

$\frac{1}{(n+1)!} \leq \frac{1}{n!}$ ✓

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ Converges by Alternate Series Test

(8) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

$a_n = \frac{1}{n!}$ $a_{n+1} = \frac{1}{(n+1)!} \leq 0.001$

$n=1$: $\frac{1}{(1+1)!} = \frac{1}{2!} = 0.5$

$n=2$: $\frac{1}{(2+1)!} = \frac{1}{3!} = 0.167$

$n=3$: $\frac{1}{(3+1)!} = \frac{1}{4!} = .042$

$n=5$: $\frac{1}{(5+1)!} = \frac{1}{6!} = .0013$

$n=6$: $\frac{1}{(6+1)!} = \frac{1}{7!} = .000198 \leq .001$ ✓

$n=6 \rightarrow$ **7 terms** b/c n starts at 0.

(9) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$ $a_n = \frac{1}{(2n+1)!}$ $a_{n+1} = \frac{1}{(2(n+1)+1)!} = \frac{1}{(2n+3)!}$

$\frac{1}{(2n+3)!} \leq 0.001$

$n=1$: $\frac{1}{(2(1)+3)!} = \frac{1}{5!} = .0083$

$n=2$: $\frac{1}{(2(2)+3)!} = \frac{1}{7!} = .000198 \leq .001$ ✓

$n=2 \downarrow$ **3 terms**

(10) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$

① $a_n = \frac{1}{(n+1)^2}$ $\lim_{n \to \infty} \frac{1}{(n+1)^2} = \frac{1}{\infty} = 0$ ✓

$\frac{1}{((n+1)+1)^2} \leq \frac{1}{(n+1)^2}$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$ converges by Alt. Series Test

② $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$ $a_n = \frac{1}{(n+1)^2}$ $b_n = \frac{1}{n^2}$

bigger

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ ← p-series test

$p=2$, converges

$\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$ converges by Direct Comparison Test

→ **ABSOLUTELY CONVERGENT**

(11) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

① $a_n = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$ $\lim_{n\to\infty} \frac{1}{n^{1/2}} = \frac{1}{\infty} = 0$ ✓

$\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges by Alt. Series Test

② $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ ← p-series ~~test~~
$p = 1/2$, diverges

∴ converges conditionally

(12) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n)}$

① $a_n = \frac{1}{\ln(n)}$ $\lim_{n\to\infty} \frac{1}{\ln(n)} = \frac{1}{\infty} = 0$ ✓

$\frac{1}{\ln(n+1)} \leq \frac{1}{\ln(n)}$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n)}$ Converges by Alt. Series Test

② $\sum_{n=1}^{\infty} \frac{1}{\ln(n)}$ $a_n = \frac{1}{\ln(n)}$ bigger $b_n = \frac{1}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ ← p-series test, $p=1$, diverges

$\sum_{n=1}^{\infty} \frac{1}{\ln(n)}$ diverges by Direct Comparison Test

∴ Converges Conditionally