AP Calculus BC
Unit 9 – Sequences & Series (Part 1)

Day 4 Notes: Direct & Limit Comparison Tests

DIRECT COMPARISON TEST Let $0 \le a_b \le b_n$ for all n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

In other words…..

1. If the larger series converges, the smaller must also converge. However, if the larger series diverges, we can't tell about the smaller one. Try a different test!

2. If the smaller series diverges, so will the larger one.

3. *Remember, when choosing a series with which to compare the given series, choose one with the same magnitude or degree.*

Examples:

1. Does $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ converge or diverge.

2. Show that $\sum_{n=2}^{\infty} \frac{1}{n-1}$ diverges.

3. Does $\sum_{n=1}^{\infty} \frac{2^n}{3^n+5}$ converge or diverge?

LIMIT COMPARISON TEST

Suppose $a_n > 0$, $b_n > 0$, and $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$, where L is <u>finite</u> and <u>positive</u>. Then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either **<u>both converge</u>** or **<u>both diverge</u>**.

Remember, when choosing a series with which to compare the given series, choose one with the same magnitude or degree.

<u>Examples</u>: Test for convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$

2. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k + 1}$

AP Calculus BC **Name: ______________________________**
Unit 9 – Day 4 – Assignment

#'s 1 – 3: Use the Direct Comparison Test to determine the convergence or divergence of the series.

1)

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

2)

$$\sum_{n=0}^{\infty} \frac{1}{3^n+1}$$

3)

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

#'s 4 – 6: Use the Limit Comparison Test to determine the convergence or divergence of the series.

4)

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

5)

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

6)

$$\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$$