**AP Calculus BC**
**Unit 9 – Day 4 – Assignment**

**Name:** Answer Key *

**#'s 1 – 3: Use the Direct Comparison Test to determine the convergence or divergence of the series.**

1)

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

$a_n = \frac{1}{n^2+1}$  $a_n = \frac{1}{n^2}$ bigger

$\sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow$ p-series test
$p = 2$
Since $p > 1$, converges

$\boxed{\therefore \sum_{n=1}^{\infty} \frac{1}{n^2+1} \text{ converges by Direct Comparison Test}}$

2)

$$\sum_{n=0}^{\infty} \frac{1}{3^n+1}$$

$a_n = \frac{1}{3^n+1}$  $b_n = \frac{1}{3^n}$ bigger

$\sum_{n=0}^{\infty} \frac{1}{3^n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$
$\downarrow$
Geometric Series Test
$r = 1/3$, since $r < 1$ converges

$\boxed{\therefore \sum_{n=0}^{\infty} \frac{1}{3^n+1} \text{ converges by Direct Comparison Test}}$

3)

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

$a_n = \frac{1}{n!}$  $b_n = \frac{1}{n^2}$ bigger

$n! = n(n-1)(n-2)(n-3)\ldots$

$\sum_{n=0}^{\infty} \frac{1}{n^2} \rightarrow$ p-series test, $p = 2$, Since $p > 1$; converges

$\boxed{\therefore \sum_{n=0}^{\infty} \frac{1}{n!} \text{ converges by Direct Comparison Test}}$

**#'s 4 – 6: Use the Limit Comparison Test to determine the convergence or divergence of the series.**

4)

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} \qquad a_n = \frac{n}{n^2+1} \qquad b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \left(\frac{n}{n^2+1} \cdot \frac{n}{1}\right) = \lim_{n\to\infty} \left(\frac{n^2}{n^2+1}\right) = 1 = L \checkmark$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \to \text{p-series test}$$
p=1, diverges

$$\boxed{\therefore \sum_{n=1}^{\infty} \frac{n}{n^2+1} \text{ diverges by Limit Comparison Test}}$$

5)

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}} \qquad a_n = \frac{1}{\sqrt{n^2+1}} \qquad b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} \cdot \frac{n}{1}\right) = \lim_{n\to\infty} \left(\frac{n}{\sqrt{n^2+1}}\right)$$

$$\frac{\frac{n}{n}}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}} \to \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = \frac{1}{\sqrt{1+0}} = L \checkmark$$

$$\sum_{n=0}^{\infty} \frac{1}{n}, \text{ p-series test}$$
p=1 → diverges

$$\boxed{\therefore \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}} \text{ diverges by Limit Comparison Test}}$$

6)

$$\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1} \qquad a_n = \frac{2n^2-1}{3n^5+2n+1} \qquad b_n = \frac{1}{n^3}$$

$$\lim_{n\to\infty} \left(\frac{2n^2-1}{3n^5+2n+1} \cdot \frac{n^3}{1}\right) = \lim_{n\to\infty} \left(\frac{2n^5-n^3}{3n^5+2n+1}\right) = \frac{2}{3} = L \checkmark$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \to \text{p-series test, } p=3\text{, converges}$$

$$\boxed{\therefore \sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1} \text{ converges by Limit Comparison test}}$$