AP Calculus BC
Unit 9 – Sequences & Series (Part 1)

Day 3 Notes: Integral & p-Series Test

The Integral Test

If f is positive, continuous, and decreasing for $x > 1$ and $a_n = f(n)$,

then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

Example 1: Does the series $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots$ converge or diverge?

First write the series in summation notation:

Now apply the integral test:

Example 2: Does $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converge or diverge?

Note: In example 2, the integral converges to $\pi/4$. This does NOT mean that $\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \pi/4$. It just means that the series converges.

p-Series and Harmonic Series

A series in the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$ is called a **p-series**.

When p = 1, we have the **harmonic series** $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$.

Convergence of p-Series

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$

Converges if $p > 1$

Diverges if $0 < p \leq 1$

Example 4:

Does $\sum_{n=1}^{\infty} \frac{n^5}{n^7}$ converge or diverge?

Your Turn #1:

Use the Integral Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{1}{n+1}$$

Your Turn #2:

Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

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Unit 9 – Day 3 – Assignment

#'s 1 – 4: Use the Integral Test to determine the convergence or divergence of the series.

1) $\sum_{n=1}^{\infty} e^{-n}$

2) $\frac{ln2}{2} + \frac{ln3}{3} + \frac{ln4}{4} + \frac{ln5}{5} + \cdots$

3) $\frac{1}{4} + \frac{2}{7} + \frac{3}{12} + \cdots + \frac{n}{n^2+3} + \cdots$

4) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

#'s 5 – 6: Determine the convergence or divergence of the p-series

5) $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$

6) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$

#'s 7 – 10: Determine the convergence or divergence of the series.

7) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$	8) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}}$
9) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$	10) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$