AP Calculus BC **Name:** Answer Key*

Unit 9 – Day 3 – Assignment

#'s 1 – 4: Use the Integral Test to determine the convergence or divergence of the series.

1) $$\sum_{n=1}^{\infty} e^{-n}$$

$\int_1^{\infty} e^{-x}\,dx$

$\lim_{b\to\infty} \int_1^b e^{-x}\,dx$

$-e^{-x}\Big]_1^b$

$-e^{-b} + e^{-1}$

$\lim_{b\to\infty}\left(\frac{-1}{e^b} + \frac{1}{e}\right) = \frac{-1}{\infty} + \frac{1}{e}$

$0 + \frac{1}{e} = \frac{1}{e}$ converges

$\boxed{\sum_{n=1}^{\infty} e^{-n} \text{ conv.}}$

2) $$\frac{ln2}{2} + \frac{ln3}{3} + \frac{ln4}{4} + \frac{ln5}{5} + \cdots$$

$\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$ $\lim_{b\to\infty} \int_1^b \frac{\ln(x+1)}{x+1}\,dx$

$u = \ln(x+1)$
$du = \frac{1}{x+1}$

$\int u\,du$

$\frac{1}{2}u^2$

$\frac{1}{2}[\ln(x+1)^2]\Big]_1^b$

$\frac{1}{2}(\ln(b+1))^2 - \frac{1}{2}(\ln(1+1))^2$

$\lim_{b\to\infty}\left(\frac{1}{2}(\ln(b+1))^2 - \frac{1}{2}(\ln 2)^2\right)$

$\infty - \frac{1}{2}(\ln 2)^2 = \infty$ diverges

$\boxed{\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1} \text{ diverges}}$

3) $$\frac{1}{4} + \frac{2}{7} + \frac{3}{12} + \cdots + \frac{n}{n^2+3} + \cdots$$

$\sum_{n=1}^{\infty} \frac{n}{n^2+3} \rightarrow \lim_{b\to\infty} \frac{1}{2}\int_1^b \frac{2x}{x^2+3}\,dx$

$u = x^2+3$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

$\frac{1}{2}\ln|x^2+3|\Big]_1^b$

$\frac{1}{2}\ln|b^2+3| - \frac{1}{2}\ln|1^2+3|$

$\lim_{b\to\infty}\left(\frac{1}{2}\ln|b^2+3| - \frac{1}{2}\ln|4|\right)$

$\infty - \frac{1}{2}\ln|4| = \infty$ div.

$\boxed{\sum_{n=1}^{\infty} \frac{n}{n^2+3} \text{ diverges}}$

4) $$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

$\lim_{b\to\infty} \int_1^b \frac{1}{x^3}\,dx$

$\int x^{-3}\,dx$

$-\frac{1}{2}x^{-2}\Big]_1^b$

$-\frac{1}{2}b^{-2} + \frac{1}{2}(1)^{-2}$

$\lim_{b\to\infty}\left(\frac{-1}{2b^2} + \frac{1}{2}\right) = \frac{-1}{\infty} + \frac{1}{2}$

$0 + \frac{1}{2} = \frac{1}{2}$ conv.

$\boxed{\sum_{n=1}^{\infty} \frac{1}{n^3} \text{ conv.}}$

#'s 5 – 6: Determine the convergence or divergence of the p-series

5) $$\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$$ $p = 5/3$

Since $p > 1$, then by p-Series test, $\boxed{\text{converges}}$

6) $$1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$$

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $p = 3/2$

Since $p > 1$, then the p-series test, $\boxed{\text{converges}}$

#'s 7 – 10: Determine the convergence or divergence of the series.

7)

$$\sum_{n=1}^{\infty} \frac{1}{2n-1}$$

$\lim_{b\to\infty} \frac{1}{2}\int_1^b \frac{2\cdot 1}{2x-1}\,dx$ $u=2x-1$, $du=2dx$

$\frac{1}{2}\int \frac{du}{u}$

$\frac{1}{2}\ln|2x-1|\Big]_1^b$

$\frac{1}{2}\ln|2b-1| - \frac{1}{2}\ln|2(1)-1|$

$\frac{1}{2}\ln|2b-1| - \cancel{\frac{1}{2}\ln 1}$

$\lim_{b\to\infty}\left(\frac{1}{2}\ln|2b-1|\right) = \infty$ diverges

$\boxed{\sum_{n=1}^{\infty} \frac{1}{2n-1} \text{ diverges}}$

8)

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}}$$

$\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ p-series $p = 5/4$

Since $p > 1$,

$\boxed{\text{converges}}$

9)

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$$

geometric series

$r = 2/3$

Since $r < 1$,

$\boxed{\text{converges}}$

10)

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$$

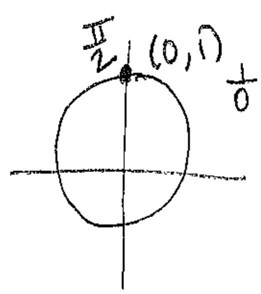
$\lim_{b\to\infty}\int_2^b \frac{1}{x\sqrt{x^2-1}}\,dx$ $u=x$, $du=dx$, $a=1$

$\int \frac{du}{u\sqrt{u^2-a^2}}$

$\text{arcsec}|x|\Big]_2^b$

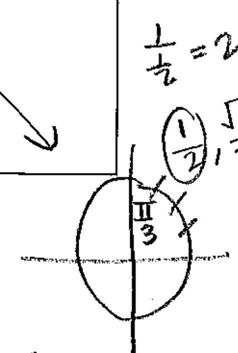
$\lim_{b\to\infty}\left(\text{arcsec}|b| - \text{arcsec}|2|\right)$

$\text{arcsec}(\infty) - \text{arcsec}\,2$



$\frac{\pi}{2} - \frac{\pi}{3}$

converges



$\boxed{\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}} \text{ converges}}$