## Day 2 Notes: Series & Convergence

### **Definition of Infinite Series:**

If  $\{a_n\}$  is an infinite sequence, then

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots + a_n + \dots$$

is an infinite series.

## **Partial Sums**

Consider the sequence of partial sums  $S_1$ ,  $S_2$ ,  $S_3$ ,...,  $S_n$ , ...

Where 
$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3$$

$$S_n=a_1+a_2+a_3+\ldots+a_n$$

If a sequence of partial sums converges, then the infinite series converges.

$$\lim_{n\to\infty} \{S_n\} = S \ then \ \sum_{n=1}^{\infty} a_n \ converges.$$

**Example 1:** Find the first four terms of the sequence of partial sums.

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

## **Geometric Series**

$$\sum_{n=0}^{\infty} ar^n, a \neq 0$$

- 1) A geometric infinite series <u>converges</u> if |r| < 1. It converges to the sum  $S = \frac{a}{1-r}$
- 2) A geometric infinite series <u>diverges</u> if  $|r| \ge 1$ .

## Example 2:

Find the sum of the series:

$$2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \cdots$$

## Example 3:

Write the decimal  $0.23\overline{23}$  as a geometric series and write its sum as the ratio of two integers.

Telescoping Series: 
$$\sum_{n=1}^{\infty} b_n - b_{n-1}$$

## Example 4:

 $\overline{\text{Let } \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n} - \frac{1}{n+1}\right)$ Find the <u>sum</u> of the telescoping series.

<b>Example 5:</b> Write the series below in telescoping form and find it	s sum.
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$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$$

# **nth Term Test for Divergence:**

If 
$$\lim_{n\to\infty} a_n \neq 0$$
, then  $\sum a_n$  diverges.

Determine whether each series converges or diverges.

## Example 6:

$$\sum_{n=1}^{\infty} \frac{n+10}{10 \ n+1}$$

## Example 7:

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^n$$

## Example 8:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

#'s 1-3: Verify that the infinite series diverges.

1)

$$\sum_{n=0}^{\infty} 1000(1.055)^n$$

2)

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

3)

$$\sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}}$$

#'s 4-5: Verify that the infinite series converges.

4)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

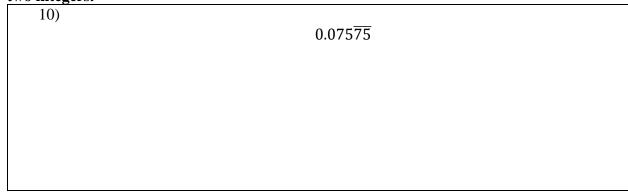
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$$\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$$

#'s $6 - 9$ : Find	the sum of the convergent seri	es.	
6)		7)	
	$\sum_{n=0}^{\infty} \frac{1}{n^2 - 1}$		$\sum_{1}^{\infty} \left(\frac{1}{2}\right)^n$
	n=2		n=0

8) 
$$1 + 0.1 + 0.01 + 0.001 + \dots$$
$$\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$$

#10: Express the repeating decimal as a geometric series, and write its sum as the ratio of two integers.



#'s 11 - 16: Determine the convergence or divergence of the series.

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11)	$\sum_{n=1}^{\infty} \frac{n+10}{5n+1}$	12)	$\sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right)$		
13)	$\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$	14)	$\sum_{n=0}^{\infty} \frac{4}{2^n}$		
15)	$\sum_{n=0}^{\infty} (1.075)^n$	16)	$\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$		