AP Calculus BC
Unit 9 – Sequences & Series (Part 1)

Day 2 Notes: Series & Convergence

Definition of Infinite Series:

If $\{a_n\}$ is an infinite sequence, then

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

is an infinite series.

Partial Sums

Consider the sequence of partial sums $S_1, S_2, S_3, \ldots, S_n, \ldots$

Where $S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$S_n = a_1 + a_2 + a_3 + \ldots + a_n$

If a sequence of partial sums converges, then the infinite series converges.

$$\lim_{n\to\infty}\{S_n\} = S \text{ then } \sum_{n=1}^{\infty} a_n \text{ converges.}$$

Example 1: Find the first four terms of the sequence of partial sums.

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Geometric Series

$$\sum_{n=0}^{\infty} ar^n, a \neq 0$$

1) A geometric infinite series **converges** if $|r| < 1$. It converges to the sum $S = \frac{a}{1-r}$
2) A geometric infinite series **diverges** if $|r| \geq 1$.

Example 2:	**Example 3:**
Find the sum of the series: $$2+\frac{3}{2}+\frac{9}{8}+\frac{27}{32}+\cdots$$	Write the decimal $0.23\overline{23}$ as a geometric series and write its sum as the ratio of two integers.

Telescoping Series:

$$\sum_{n=1}^{\infty} b_n - b_{n-1}$$

Example 4:

Let $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n-1}-\frac{1}{n}\right)+\left(\frac{1}{n}-\frac{1}{n+1}\right)$

Find the **sum** of the telescoping series.

Example 5: Write the series below in telescoping form and find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$$

nth Term Test for Divergence:

$$\boldsymbol{If\ \lim_{n\to\infty} a_n \neq 0,\ then\ \sum a_n diverges.}$$

Determine whether each series converges or diverges.

Example 6:

$$\sum_{n=1}^{\infty} \frac{\boldsymbol{n+10}}{\boldsymbol{10\,n+1}}$$

Example 7:

$$\sum_{n=1}^{\infty} \boldsymbol{3\left(\frac{2}{3}\right)^{n}}$$

Example 8:

$$\sum_{n=1}^{\infty} \frac{\boldsymbol{1}}{\boldsymbol{n(n+3)}}$$

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Unit 9 – Day 2 – Assignment

#'s 1 – 3: Verify that the infinite series diverges.

1) $\displaystyle\sum_{n=0}^{\infty} 1000(1.055)^n$	2) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$
3) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}}$	

#'s 4 – 5: Verify that the infinite series converges.

4) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$	5) $\displaystyle\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$

#'s 6 – 9: Find the sum of the convergent series.

6) $$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$	7) $$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$
8) 1 + 0.1 + 0.01 + 0.001 + ….	9) $$\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$$

#10: Express the repeating decimal as a geometric series, and write its sum as the ratio of two integers.

10)

$$0.075\overline{75}$$

#'s 11 - 16: Determine the convergence or divergence of the series.

11) $$\sum_{n=1}^{\infty} \frac{n+10}{5n+1}$$	**12)** $$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$$
13) $$\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$$	**14)** $$\sum_{n=0}^{\infty} \frac{4}{2^n}$$
15) $$\sum_{n=0}^{\infty} (1.075)^n$$	**16)** $$\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$$