AP Calculus BC **Name:** Answer Key*

Unit 9 – Day 2 – Assignment

#'s 1 – 3: Verify that the infinite series diverges.

1)

$$\sum_{n=0}^{\infty} 1000(1.055)^n$$

geometric series

$r = 1.055$

$\boxed{\text{diverges}}$ b/c $r \geq 1$

2)

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

$\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$

$\boxed{\text{diverges}}$

b/c of nth term test

3)

$$\sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}}$$

$$\frac{\frac{2^n}{2^{n+1}} + \frac{1}{2^{n+1}}}{\frac{2^{n+1}}{2^{n+1}}} = \frac{\frac{1}{2} + \frac{1}{2^{n+1}}}{1}$$

$\lim_{n\to\infty} \frac{1}{2} + \frac{1}{2^{n+1}} = \frac{1}{2} + \frac{1}{\infty}$

$= \frac{1}{2} \neq 0$

$\boxed{\text{Diverges}}$ b/c of nth term test

#'s 4 – 5: Verify that the infinite series converges.

4)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$1 = A(n+1) + B(n)$

$n=-1$: $1 = A\cancel{(-1+1)} + B(-1)$

$B = -1$

$n=0$: $1 = A(0+1) + \cancel{B(0)}$

$A = 1$

$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

Telescoping series

$= \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right)$

$+ \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \ldots$

$+ \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$

$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1 - 0$

$= \boxed{1}$

$\boxed{\text{converges}}$

5)

$$\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$$

geometric series

$r = 3/4$

$\boxed{\text{converges}}$ since $r < 1$

$S = \frac{2}{1 - \frac{3}{4}} = \frac{2}{\frac{1}{4}} = 2.4$

$= \boxed{8}$

#'s 6 – 9: Find the sum of the convergent series.

6)

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

Telescoping series

$(n+1)(n-1)$

$\frac{1}{n^2-1} = \frac{A^{(n-1)}}{n+1} + \frac{B^{(n+1)}}{n-1}$

$(n-1) \quad (n+1)$

$1 = A(n-1) + B(n+1)$

$n=1$: $1 = A\cancel{(1-1)} + B(1+1)$

$B = 1/2$

$n=-1$: $1 = A(-1-1) + \cancel{B(-1+1)}$

$-1/2 = A$

$\frac{1}{2}\sum_{n=2}^{\infty} \frac{1}{n-1} - \frac{1}{n+1}$

$= \frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \ldots + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right)\right]$

$\lim_{n \to \infty} \left[\frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1}\right)\right]$

$= \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{1}{2} \cdot \frac{3}{2} = \boxed{\frac{3}{4}}$

7)

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

geometric series

$r = 1/2$

converges

$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = \boxed{2}$

8)

$1 + 0.1 + 0.01 + 0.001 + \ldots$

$\frac{.1}{1} = .1$

$r = .1$ geometric series

converges

$S = \frac{1}{1-.1} = \frac{1}{.9} = \frac{1}{\frac{9}{10}} = \boxed{\frac{10}{9}}$

9)

$$\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$$

$\sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n}$

geometric $\left(\frac{1}{2}\right)^n$

$r = 1/2$

$S = \frac{1}{1-1/2} = 2$

geom $\left(\frac{1}{3}\right)^n$

$r = 1/3$ $S = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$2 - \frac{3}{2} = \boxed{\frac{1}{2}}$

#10: Express the repeating decimal as a geometric series, and write its sum as the ratio of two integers.

10)

$0.07\overline{575}$

$.075 + .00075 + .0000075 + \ldots$

$\frac{.00075}{.075} = .01 = r$

$\boxed{\sum_{n=1}^{\infty} .075(.01)^n}$

Converges b/c $r < 1$

$S = \frac{.075}{1-.01} = \frac{.075^{(1000)}}{.99_{(1000)}} = \frac{75}{990} = \boxed{\frac{5}{66}}$

#'s 11 - 16: Determine the convergence or divergence of the series.

11)

$$\sum_{n=1}^{\infty} \frac{n+10}{5n+1}$$

$\lim_{n\to\infty} \frac{n+10}{5n+1} = \frac{1}{5} \neq 0$

$\boxed{\text{diverges}}$

b/c of nth term test

12)

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$$

Telescoping Series

$\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right)$

$+ \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \ldots \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$\lim_{n\to\infty} \left(1 + \frac{1}{2} - \frac{1}{n+2}\right) = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$

$\boxed{\text{Converges}}$

13)

$$\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$$

$\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{3}{2} \neq 0$

$\boxed{\text{diverges}}$

b/c of nth term test

14)

$$\sum_{n=0}^{\infty} \frac{4}{2^n}$$ geom series

$\sum_{n=0}^{\infty} 4\left(\frac{1}{2}\right)^n$ $r = 1/2$

$r > 1$ convg

$\frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = \boxed{8}$ $\boxed{\text{Converges}}$

15)

$$\sum_{n=0}^{\infty} (1.075)^n$$

geom. series

$r = 1.075$

$\boxed{\text{diverges}}$ b/c $r > 1$

16)

$$\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$$

$\lim_{n\to\infty} \frac{n}{\ln(n)} = \lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} (n)$

$= \infty \neq 0$

$\boxed{\text{diverges}}$

$\boxed{n^{th} \text{ term test}}$