AP Calculus BC
Unit 9 – Sequences & Series (Part 1)

Day 1 Notes: Sequences

A **sequence** is a list of terms: $a_1, a_2, a_3, \ldots, a_n, \ldots$
a_1 = first term
a_n = nth term
$\{a_n\} = a_1, a_2, a_3, \ldots, a_n, \ldots$

Limit of a Sequence:

Let f be a function of real variables such that $\lim_{x \to \infty} f(x) = L$. If $\{a_n\}$ is a sequence such that f(n) = a_n for every positive integer n, then $\lim_{n \to \infty} a_n = L$.

Example 1: Write out the first four terms of the sequence and then find the limit of the sequence with the nth term: $a_n = \frac{\ln n^2}{n}$	**Example 2:** Find the limit of $\{a_n\} = \frac{(n+1)!}{n!}$
Example 3: Find the limit of $\{a_n\} = \{3 + (-1)^n\}$	**Example 4:** Find the limit of $\{b_n\} = \left\{\frac{n}{1-2n}\right\}$

Monotonic Sequence:

$\{a_n\}$ is monotonic if

$a_1 \le a_2 \le a_3 \le \ldots \le a_n \le \ldots$ (increasing) OR

$a_1 \ge a_2 \ge a_3 \ge \ldots \ge a_n \ge \ldots$ (decreasing)

Bounded Sequence:

1) $\{a_n\}$ is bounded above if $a_n \le M$.
2) $\{a_n\}$ is bounded below if $a_n \ge N$.
3) $\{a_n\}$ is bounded if $N \le a_n \le M$

If a sequence is bounded and monotonic, then it will converge.

Example 5: Show that a_n is bounded and monotonic. $\{a_n\} = \left\{\frac{1}{n}\right\}$	**Example 6:** Show that a_n is monotonic and bounded below. $\{a_n\} = \left\{\frac{n^2}{n+1}\right\}$

Absolute Value Theorem:

If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

Example 7:

Show that $\lim_{n\to\infty} a_n = 0$ if $\{a_n\} = \left\{\frac{(-1)^n n}{3^n}\right\}$

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Unit 9 – Day 1 – Assignment

#'s 1 – 2: Write the first five terms of the sequence.

1) $a_n = 5 - \frac{1}{n} + \frac{1}{n^2}$	2) $a_n = \frac{3^n}{n!}$

3) Write the next two apparent terms of the sequence. Describe the pattern you used to find these terms.

$\frac{7}{2}, 4, \frac{9}{2}, 5, \ldots$

#'s 4 – 5: Simplify the ratio of factorials.

4) $\frac{10!}{8!}$	5) $\frac{(2n-1)!}{(2n+1)!}$

#'s 6 – 15: Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

6) $a_n = \frac{5n^2}{n^2+2}$	7) $a_n = \frac{2n}{\sqrt{n^2+1}}$
8) $a_n = sin\frac{1}{n}$	9) $a_n = \frac{n+1}{n}$

10) $$a_n = cos\frac{n\pi}{2}$$	11) $$a_n = (-1)^n\left(\frac{n}{n+1}\right)$$
12) $$a_n = \frac{1 + (-1)^n}{n}$$	13) $$a_n = \frac{3^n}{4^n}$$
14) $$a_n = \frac{n-1}{n} - \frac{n}{n-1}, n \geq 2$$	15) $$a_n = \frac{n^p}{e^n}, p > 0$$

#'s 16 – 18: Determine whether the sequence with the given nth term is monotonic. Discuss the boundedness of the sequence.

16) $$a_n = 4 - \frac{1}{n}$$	17) $$a_n = \frac{n}{2^{n+2}}$$
18) $$a_n = sin\frac{n\pi}{6}$$	