AP Calculus BC — **Name:** Answer Key *

Unit 9 – Day 1 – Assignment

#'s 1 – 2: Write the first five terms of the sequence. $\boxed{3, \frac{9}{2}, \frac{9}{2}, \frac{27}{8}, \frac{81}{40}}$

1) $a_n = 5 - \frac{1}{n} + \frac{1}{n^2}$

$a_1: 5 - \frac{1}{1} + \frac{1}{1^2} = 5$

$a_2: 5 - \frac{1}{2} + \frac{1}{2^2} = \frac{19}{4}$

$a_3: 5 - \frac{1}{3} + \frac{1}{3^2} = \frac{43}{9}$

$a_4: 5 - \frac{1}{4} + \frac{1}{4^2} = \frac{77}{16}$

$a_5: 5 - \frac{1}{5} + \frac{1}{5^2} = \frac{121}{25}$

$\boxed{5, \frac{19}{4}, \frac{43}{9}, \frac{77}{16}, \frac{121}{25}}$

2) $a_n = \frac{3^n}{n!}$

$a_1: \frac{3^1}{1!} = 3$

$a_2: \frac{3^2}{2!} = \frac{9}{2}$

$a_3: \frac{3^3}{3!} = \frac{27}{6} = \frac{9}{2}$

$a_4: \frac{3^4}{4!} = \frac{81}{24} = \frac{27}{8}$

$a_5: \frac{3^5}{5!} = \frac{243}{120} = \frac{81}{40}$

3) Write the next two apparent terms of the sequence. Describe the pattern you used to find these terms.

$\frac{7}{2}, 4, \frac{9}{2}, 5, \ldots$

$\frac{7}{2}, \frac{8}{2}, \frac{9}{2}, \frac{10}{2}, \frac{11}{2}, \frac{12}{2}$

$a_1 \quad a_2 \quad a_3 \quad a_4$

$\boxed{\frac{11}{2}, 6}$

$\boxed{a_n = \frac{n+6}{2}}$

#'s 4 – 5: Simplify the ratio of factorials.

4) $\frac{10!}{8!}$

$\frac{10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!}} = 10(9) = \boxed{90}$

5) $\frac{(2n-1)!}{(2n+1)!}$

$\frac{\cancel{(2n-1)!}}{(2n+1)(2n)\cancel{(2n-1)!}} = \boxed{\frac{1}{2n(2n+1)}}$

#'s 6 – 15: Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

6) $a_n = \frac{5n^2}{n^2+2}$

$\lim_{n\to\infty} \frac{5n^2}{n^2+2} = \boxed{5}$ $\boxed{\text{converges}}$

H.A → Look @ leading coeff.

7) $a_n = \frac{2n}{\sqrt{n^2+1}}$ $\boxed{\text{Converges}}$

$\lim_{n\to\infty} \frac{2n}{\sqrt{n^2+1}}$

Divide by n → $\frac{\frac{2n}{n}}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}}$

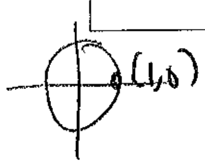
$\lim_{n\to\infty} \frac{2}{\sqrt{1+\frac{1}{n^2}}} = \frac{2}{\sqrt{1+\frac{1}{\infty}}} = \frac{2}{\sqrt{1}} = \boxed{2}$

8) $a_n = sin\frac{1}{n}$

$\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = \sin\left(\frac{1}{\infty}\right)$

$= \sin(0)$

$= \boxed{0}$



$\boxed{\text{Converges}}$

9) $a_n = \frac{n+1}{n}$

$\lim_{n\to\infty} \left(\frac{n+1}{n}\right) = \boxed{1}$

H.A. → Look @ leading coeff.

$\boxed{\text{Converges}}$

$\boxed{\text{diverges}}$ b/c oscillates between positive & negative.

10)

$$a_n = cos\frac{n\pi}{2}$$

$\lim_{n\to\infty} \cos\left(\frac{n\pi}{2}\right) = \cos\left(\frac{\infty\pi}{2}\right)$

$= \cos(\infty) = DNE$

$\boxed{\text{diverges}}$

11)

$$a_n = (-1)^n\left(\frac{n}{n+1}\right)$$

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$

you can't use abs. value Thm

$a_1 : (-1)^1\left(\frac{1}{1+1}\right) = -1/2$

$a_2 : (-1)^2\left(\frac{2}{2+1}\right) = 2/3$

$a_3 : (-1)^3\left(\frac{3}{3+1}\right) = -\frac{3}{4}$

12)

$$a_n = \frac{1 + (-1)^n}{n}$$

$\lim_{n\to\infty} \frac{1+(-1)^n}{n} = \frac{\#}{\infty} = \boxed{0}$

$a_1 : 0$
$a_2 : 1$

$a_3 : 0$ $a_5 : 0$

$a_4 : 1/2$ $a_6 : 1/3$

$\boxed{\text{Converges}}$

13)

$$a_n = \frac{3^n}{4^n}$$

$\lim_{n\to\infty}\left(\frac{3}{4}\right)^n = \left(\frac{3}{4}\right)^\infty = \boxed{0}$

$\boxed{\text{converges}}$

14)

$$a_n = \frac{(n-1)}{(n-1)}\frac{n-1}{n} + \frac{-n}{n-1}\frac{(n)}{(n)}, n \geq 2$$

$\frac{n^2-2n+1-n^2}{n(n-1)} = \frac{-2n+1}{n(n-1)}$

$\lim_{n\to\infty} \frac{-2n+1}{n^2-n} = \lim_{n\to\infty} \frac{-2}{2n-1} = \frac{-2}{\infty} = \boxed{0}$

$\boxed{\text{converges}}$

15)

$$a_n = \frac{n^p}{e^n}, p > 0$$

$\lim_{n\to\infty} \frac{n^p}{e^n} = \frac{\#}{e^\infty} = \frac{\#}{\infty} = \boxed{0}$ $\boxed{\text{converges}}$

#'s 16 – 18: Determine whether the sequence with the given nth term is monotonic. Discuss the boundedness of the sequence.

16)

$$a_n = 4 - \frac{1}{n}$$

$4 - \frac{1}{n+1}$

$a_1 : 4 - \frac{1}{1} = 3$

$a_2 : 4 - \frac{1}{2} = 3.5$

$a_3 : 4 - \frac{1}{3} = \frac{11}{3} = 3.\overline{6}$

$a_4 : 4 - \frac{1}{4} = 3.75$

increasing $\boxed{\text{, monotonic}}$

$\lim_{n\to\infty}\left(4 - \frac{1}{n}\right) = 4 - \frac{1}{\infty} = 4$

$\boxed{\text{Bounded by 3 \& 4}}$

17)

$$a_n = \frac{n}{2^{n+2}}$$

$a_1 : \frac{1}{2^3} = \frac{1}{8}$

$a_2 : \frac{2}{2^4} = \frac{2}{16}$

$a_3 : \frac{3}{2^5} = \frac{3}{32}$

decreasing

$\boxed{\text{monotonic}}$

$\frac{n+1}{2^{n+2+1}} = \frac{n+1}{2^{n+3}}$

increasing faster

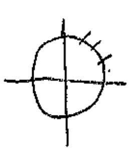
$\lim_{n\to\infty} \frac{n}{2^{n+2}}$

$\lim_{n\to\infty} \frac{1}{(n2)(2^{n+2})(1)} = \frac{1}{\infty} = 0$

$\boxed{\text{bounded by } \frac{1}{8} \text{ \& } 0}$

18)

$$a_n = sin\frac{n\pi}{6}$$



$a_1 : \sin\left(\frac{\pi}{6}\right) = 1/2$

$a_2 : \sin\left(\frac{2\pi}{6}\right) = \sqrt{3}/2$

$a_3 : \sin\left(\frac{3\pi}{6}\right) = 1$

$a_4 : \sin\left(\frac{4\pi}{6}\right) = \sqrt{3}/2$

$\boxed{\text{not monotonic}}$

sin graphs are bounded by -1 & 1

$\boxed{\text{bounded by } -1 \text{ \& } 1}$