Unit 8 Review – Scavenger Hunt

A. $\frac{1}{3}\int \sqrt{3x-1}\, dx\,(3)$

$u = 3x-1$

$du = 3dx$

$\frac{1}{3}\int \sqrt{u}\, du$

$\frac{1}{3}\int u^{1/2}\, du$

$\frac{1}{3}\left[\frac{2}{3}u^{3/2}\right] + C$

$\frac{2}{9}u^{3/2} + C = \boxed{\frac{2}{9}(3x-1)^{3/2} + C}$

O. $\int \frac{x}{\sqrt{2x-1}}\, dx$

$u = 2x-1$

$du = 2dx$

$\frac{du}{2} = dx$

$\int x(2x-1)^{-1/2}\, dx$

$\frac{u+1}{2} = x$

$\int \left(\frac{u+1}{2}\right)(u)^{-1/2}\left(\frac{du}{2}\right)$

$\frac{1}{4}\int (u+1)(u^{-1/2})\, du$

$\frac{1}{4}\int u^{1/2} + u^{-1/2}\, du$

$\frac{1}{4}\left[\frac{2}{3}u^{3/2} + \frac{2}{1}u^{1/2}\right] + C$

$\frac{1}{6}u^{3/2} + \frac{1}{2}u^{1/2} + C$

$\boxed{\frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{1/2} + C}$

(J.) $\int \frac{dx}{9+x^2}$

$u=x$
$du=dx$
$a=3$

$\int \frac{du}{a^2+u^2}$

$\boxed{\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C}$

(F.) $\frac{1}{3}\int \frac{3x^2 \, dx}{\sqrt{25-x^6}}$

$u=x^3$
$du=3x^2dx$
$a=5$

$\frac{1}{3}\int \frac{du}{\sqrt{a^2-u^2}}$

$\boxed{\frac{1}{3}\arcsin\left(\frac{x^3}{5}\right)+c}$

(B.) $\int xe^{3x}\,dx$

$u=x$
$du=dx$

$v=\frac{1}{3}e^{3x}$
$dv=e^{3x}dx$

$(x)\left(\frac{1}{3}e^{3x}\right) - \int\left(\frac{1}{3}e^{3x}\right)(dx)$

$-\frac{1}{3}\int e^{3x}\,dx$

$-\frac{1}{3}\left[\frac{1}{3}e^{3x}\right]+C$

$\boxed{\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}+C}$

K. $\int \cos^{3}x \sin^2 x \, dx$ odd

$\int \cos^2 x \sin^2 x \cos x \, dx$

$\int (1-\sin^2 x)(\sin^2 x)(\cos x) \, dx$

$\int (\sin^2 x - \sin^4 x) \cos x \, dx$

$\int u^2 - u^4 \, du$

$u = \sin x$
$du = \cos x \, dx$

$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$\boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

I. $\int \sec^{4} x \tan x \, dx$ even

$\frac{\sin^2 x + \cos^2 x = 1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

$\int \sec^2 x \tan x \sec^2 x \, dx$

$\int (\tan^2 x + 1)(\tan x) \sec^2 x \, dx$

$\int (\tan^3 x + \tan x)(\sec^2 x) \, dx$

$u = \tan x$
$du = \sec^2 x \, dx$

$\int u^3 + u \, du$

$\frac{1}{4}u^4 + \frac{1}{2}u^2 + C$

$\boxed{\frac{1}{4}\tan^4 x + \frac{1}{2}\tan^2 x + C} = \frac{1}{4}\sec^4 x + C$

(E.) $\int \frac{1}{x^2+x}\,dx$

$\uparrow$

Factor: $x^2+x = x(x+1)$

$$\frac{1}{x^2+x} = \frac{A^{(x+1)}}{x_{(x+1)}} + \frac{B^{(x)}}{x+1_{(x)}}$$

$1 = A(x+1) + B(x)$

$\underline{x=-1}$: $1 = A(\cancel{-1+1}) + B(-1)$

$B=-1$

$\underline{x=0}$: $1 = A(0+1) + \cancel{B(0)}$

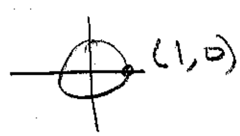
$1 = A$

$$\int \frac{1}{x}\,dx + \int \frac{-1}{x+1}\,dx$$

$$\boxed{\ln|x| - \ln|x+1| + C}$$

(C) $\int_0^{\infty} \cos\frac{x}{2}\,dx$

$\lim_{b\to\infty} \int_0^b \cos\left(\frac{x}{2}\right)dx$



$\left[2\sin\left(\frac{x}{2}\right)\right]_0^b$

$2\sin\left(\frac{b}{2}\right) - 2\sin\left(\frac{0}{2}\right)$

$2\sin\left(\frac{b}{2}\right) - 0$

$\lim_{b\to\infty} 2\sin\left(\frac{b}{2}\right) = \text{DNE}$ $\boxed{\text{diverges}}$

6. $\frac{dy}{dt} = \frac{3y}{5}\left(1 - \frac{y}{5}\right)$; $y(0)=2$

$\frac{dy}{dt} = \frac{3}{5}y(5-y)$

$\int \frac{dy}{y(5-y)} = \int \frac{3}{5}\,dt$

$\frac{1}{y(5-y)} = \frac{A}{y} + \frac{B}{5-y}$

$1 = A(5-y) + B(y)$

$\underline{y=5}$: $1 = A\cancel{(5-5)} + B(5)$

$B = 1/5$

$\underline{y=0}$: $1 = A(5-0) + \cancel{B(0)}$

$A = 1/5$

$\int \frac{1/5}{y} + \frac{1/5}{5-y}\,dy = \int \frac{3}{5}\,dt$

$\frac{1}{5}\ln|y| - \frac{1}{5}\ln|5-y| = \frac{3}{5}t + C$

$\frac{\cancel{\frac{1}{5}}(\ln y - \ln(5-y))}{\cancel{\frac{1}{5}}} = \frac{\frac{3}{5}t + C}{\frac{1}{5}}$

$\ln y - \ln(5-y) = 3t + C$

$-\ln y + \ln(5-y) = -3t - C$

$\cancel{e^{\ln}}\left(\frac{5-y}{y}\right) = e^{-3t-C}$

$\frac{5-y}{y} = e^{-3t}e^{-C}$

$\frac{5}{y} - 1 = e^{-3t}e^{-C}$

$\frac{5}{y} = 1 + e^{-3t}e^{-C}$

$y(0) = 2$

$\frac{5}{2} = 1 + e^{-3(0)}e^{-C}$

$\frac{5}{2} = 1 + (1)(e^{-C})$

$1.5 = e^{-C}$

$\frac{5}{y} = 1 + 1.5e^{-3t}$

$5 = y(1 + 1.5e^{-3t})$

$$\boxed{y = \frac{5}{1 + 1.5e^{-3t}}}$$

L. $\frac{1}{5}\int \sec 5x \tan 5x \, dx \, (5)$

$\frac{1}{5}\int \sec u \tan u \, du$ $\quad u = 5x$, $du = 5\,dx$

$\frac{1}{5}[\sec u] + C$

$\boxed{\frac{1}{5}\sec 5x + C}$

N. $\int \frac{dx}{x^2+12x+45}$

can't factor denominator → complete the square

$\int \frac{dx}{(x+6)^2+9}$ $\quad x^2+12x+\left(\frac{12}{2}\right)^2+45-\left(\frac{12}{2}\right)^2$

$(x+6)^2 + 9$

$\int \frac{du}{u^2+a^2}$ $\quad u = x+6$, $du = dx$, $a = 3$

$\boxed{\frac{1}{3}\arctan\left(\frac{x+6}{3}\right) + C}$

D.) $\int \frac{4x^2}{x^2+25}\,dx$

$$\begin{array}{r} 4 \\ x^2+0x+25 \enclose{longdiv}{4x^2+0x+0} \\ \ominus\; \cancel{4x^2} \ominus \cancel{0x} \ominus 100 \\ \hline -100 \end{array}$$

$$\int 4\,dx + \int \frac{-100}{x^2+25}\,dx$$

$4x$ $\quad -100\int \frac{du}{u^2+a^2}$

$u = x$
$du = dx$
$a = 5$

$$-100\left(\tfrac{1}{5}\right)\arctan\left(\tfrac{x}{5}\right)$$

$$\boxed{4x - 20\arctan\left(\tfrac{x}{5}\right) + C}$$

H.) $\int \ln 8x\,dx$

$u = \ln 8x \qquad v = x$
$du = \frac{8}{8x}\,dx \qquad dv = 1\,dx$

$$(\ln 8x)(x) - \int (x)\left(\frac{8}{8x}\right)dx$$

$$-\int dx$$

$$-x$$

$$\boxed{x\ln 8x - x + C}$$

M. $\int_2^{\infty} \frac{1}{(x-1)^2}\, dx$

$\lim_{b\to\infty} \int_2^{b} \frac{1}{(x-1)^2}\, dx$

$u = x+1$
$du = dx$

$\int u^{-2}\, du$

$-1u^{-1}$

$\left[\frac{-1}{x-1}\right]_2^b$

$\frac{-1}{b-1} - \frac{-1}{2-1} = \frac{-1}{b-1} - \frac{-1}{1}$

$\lim_{b\to\infty}\left[\frac{-1}{b-1} + 1\right]$

$\frac{-1}{\infty} + 1 = 0 + 1$

$\boxed{\text{Converges to } 1}$