Unit 8 Review
BINGO

$$\int x\csc x\cot x\,dx$$

$u = x$ $\qquad v = -\csc x$

$du = dx$ $\qquad dv = \csc x\cot x\,dx$

$(x)(-\csc x) - \int(-\csc x)\,dx$

$+ \int \csc x\,dx$

$+ -\ln|\csc x + \cot x| + C$

$$\boxed{-x\csc x - \ln|\csc x + \cot x| + C}$$

If P(0) = 10, what is the $\lim_{t\to\infty} P(t)$?

$$\frac{dP}{dt} = 5y(18-y)$$

$\boxed{18}$

$x^2 - 6x + \left(\frac{-6}{2}\right)^2 + 9 - \left(\frac{-6}{2}\right)^2$

$(x-3)^2$

$$\int \frac{3x}{x^2-6x+9}\,dx$$

$(x-3)(x-3)$

$\frac{3x}{(x-3)^2} = \frac{A}{(x-3)}^{(x-3)} + \frac{B}{(x-3)}^{(x-3)}$

$x=2$: $3(2) = A(2-3) + B(2-3)$

$6 = -A - B$

$x=1$: $3(1) = A(1-3) + B(1-3)$

$3 = -2A - 2B$

$12 = -\cancel{2}A - 2B$

$\ominus\ 3 = -\cancel{2}A - 2B$

$9 =$

$$\int \sin^2 x \cos^2 x \, dx$$

$\int \frac{1}{2}(1-\cos 2x) \cdot \frac{1}{2}(1+\cos 2x)\,dx$

$\frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\,dx$

$\frac{1}{4}\int 1 - \cos^2 2x \, dx$

$\frac{1}{4}\int 1 - \left[\frac{1}{2}(1+\cos 4x)\right] dx$

$\frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$

$\frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos 4x \, dx$

$\frac{1}{4}\int \frac{1}{2}dx - \frac{1}{8}\int \cos 4x \, dx$

$\frac{1}{4}\left(\frac{1}{2}x\right) - \frac{1}{8}\left[\frac{1}{4}\sin 4x\right]$

$\frac{1}{8}x - \frac{1}{32}\sin 4x + C$

$\boxed{\frac{1}{32}(4x - \sin 4x) + C}$

Solve the differential equation.

$$\frac{dy}{dt} = 3y(4-y)$$

$$y(0) = 8$$

$\int \frac{dy}{y(4-y)} = \int 3\,dt$

$\frac{1}{y(4-y)} = \frac{A}{y} + \frac{B}{4-y}$

$1 = A(4-y) + B(y)$

$1 = B(4)$

$B = 1/4 \quad A = 1/4$

$\int \frac{1/4}{y} + \int \frac{1/4}{4-y}\,dy = \int 3\,dt$

$\frac{1}{4}\ln|y| - \frac{1}{4}\ln|4-y| = 3t + C$

$\frac{1}{4}(\ln y - \ln(4-y)) = 3t + C$

$\ln y - \ln(4-y) = 12t + C$

$\ln\left(\frac{4-y}{y}\right) = -12t - C$

$\frac{4}{y} - 1 = e^{-12t}e^{-C}$

$\frac{4}{y} = 1 + e^{-12t}e^{-C}$

$\frac{4}{8} = 1 + \cancel{e^{-12t}}^{\,1} e^{-C}$

$-.5 = e^{-C}$

$\boxed{y = \frac{4}{1 - .5e^{-12t}}}$

$$\int \frac{e^x}{1+e^x}\,dx$$

$u = 1 + e^x$

$du = e^x dx$

$\int \frac{du}{u}\,dx$

$\ln|u| + C$

$\ln|1+e^x| + C$

$\boxed{\ln(1+e^x) + C}$

$$\int_0^4 \frac{1}{\sqrt{x}}\,dx$$

$\lim_{b \to 0} \int_b^4 x^{-1/2}\,dx$

$2x^{1/2}\Big]_b^4 = 2(4)^{1/2} - 2(b)^{1/2}$

$2\sqrt{4} - 2\sqrt{b}$

$\lim_{b\to 0}\left[4 - 2\sqrt{b}\right]$

$4 - 2\sqrt{0} = 4$

$\boxed{\text{Converges to } 4}$

$$\frac{1}{4\pi}\int^{4\pi} x(\cos 2\pi x^2)\,dx$$

$u = 2\pi x^2$
$du = 4\pi x\,dx$

$\frac{1}{4\pi}\int \cos u\,du$

$\frac{1}{4\pi}[\sin u] + C$

$\boxed{\frac{1}{4\pi}\sin 2\pi x^2 + C}$

$$\int \frac{1}{x^2-4}\,dx$$

$(x+2)(x-2)$

$\frac{1}{x^2-4} = \frac{A(x-2)}{x+2 \; (x-2)} + \frac{B(x+2)}{x-2 \; (x+2)}$

$1 = A(x-2) + B(x+2)$

$\underline{x=2}$: $1 = A(\cancel{2-2}) + B(2+2)$

$B = 1/4$

$\underline{x=-2}$: $1 = A(-2-2) + B(\cancel{-2+2})$

$A = -1/4$

$\int \frac{-\frac{1}{4}}{x+2} + \int \frac{1/4}{x-2}$

$-\frac{1}{4}\ln|x+2| + \frac{1}{4}\ln|x-2| + C$

$\boxed{\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C}$

$$\int_1^{\infty} \frac{1}{x^2}\,dx$$

$p = 2$ converges $\frac{1}{p-1}$

$\frac{1}{2-1} = 1$

$\boxed{\text{Converges to } 1}$

$$\int xe^{-2x}\,dx$$

$u = x$ $\quad v = -\frac{1}{2}e^{-2x}$
$du = dx$ $\quad dv = e^{-2x}dx$

$(x)(-\frac{1}{2}e^{-2x}) - \int(-\frac{1}{2}e^{-2x})dx$

$+\frac{1}{2}\int e^{-2x}dx$

$\frac{1}{2}\left[-\frac{1}{2}e^{-2x}\right]$

$-\frac{1}{4}e^{-2x}$

$-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$

$\boxed{-\frac{1}{4e^{2x}}(2x+1)+C}$

$$\frac{1}{3}\int 3x^2 e^{x^3}\, dx$$

$u = x^3$
$du = 3x^2 dx$

$\frac{1}{3}\int e^u \, du$

$\frac{1}{3}e^u + C$

$\boxed{\frac{1}{3}e^{x^3} + C}$

What are the horizontal asymptotes?

$$\frac{dy}{dx} = y(7 - 0.001y)$$

$.001y(7000 - y)$

$\uparrow$ carrying capacity

$\boxed{0 \text{ \& } 7000}$

$$\int e^{5x}\, dx$$

$\boxed{\frac{1}{5}e^{5x} + C}$

$$\int sec^2 x tan x \, dx$$

$u = \tan x$
$du = \sec^2 x\, dx$

$\int u \, du$

$\frac{1}{2}u^2 + C$

$\boxed{\frac{1}{2}\tan^2 x + C}$

$$-\int -\cos^3 x \sin x \, dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-\int u^3\,du$

$-\frac{1}{4}u^4 + C$

$\boxed{-\frac{1}{4}\cos^4 x + C}$

$$\frac{1}{3}\int 3\sec 3x\,dx$$

$u = 3x$

$du = 3dx$

$\frac{1}{3}\int \sec u\,du$

$\frac{1}{3}\ln|\sec u + \tan u|$

$\boxed{\frac{1}{3}\ln|\sec 3x + \tan 3x| + C}$

$$\int \frac{3x+2}{x^2+9}\,dx$$

$u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}\cdot 3\int \frac{2\cancel{3}x}{x^2+9}\,dx + \int \frac{2}{x^2+9}\,dx$

$\frac{3}{2}\int \frac{du}{u}$

$u = x$

$du = dx$

$a = 3$

$\frac{3}{2}\ln|x^2+9| + \frac{2}{3}\arctan\left(\frac{x}{3}\right) + C$

$\boxed{\frac{3}{2}\ln(x^2+9) + \frac{2}{3}\arctan\left(\frac{x}{3}\right) + C}$

$$\frac{1}{3}\int 3x^2\sqrt{x^3-1}\,dx$$

$u = x^3-1$

$du = 3x^2dx$

$\frac{1}{3}\int u^{1/2}\,du$

$\frac{1}{3}\left[\frac{2}{3}u^{3/2}\right] + C$

$\boxed{\frac{2}{9}(x^3-1)^{3/2} + C}$

What is the carrying capacity?

$$\frac{dy}{dt} = 4y(6 - 2y)$$

(2) $\frac{6}{2}$ $\frac{2y}{2}$

$8y(3-y)$

$\boxed{3}$

$$\int e^{2x} \sin x \, dx$$

$u = \sin x$ $\quad v = \frac{1}{2}e^{2x}$

$du = \cos x$ $\quad dv = e^{2x}dx$

$(\sin x)(\frac{1}{2}e^{2x}) - \int (\frac{1}{2}e^{2x})(\cos x)$

$-\frac{1}{2}\int e^{2x}\cos x$

$u = \cos x$ $\quad v = \frac{1}{2}e^{2x}$

$du = -\sin x$ $\quad dv = e^{2x}$

$-\frac{1}{2}[(\cos x)(\frac{1}{2}e^{2x}) - \int (\frac{1}{2}e^{2x})(-\sin x)$

$-\frac{1}{4}\cos x e^{2x}$ $\quad + \frac{1}{2}\int \frac{1}{2}e^{2x}(-\sin x)$

$-\frac{1}{4}\int e^{2x}\sin x$

$\int e^{2x}\sin x\,dx = \frac{1}{2}\sin x e^{2x} - \frac{1}{4}\cos x e^{2x} - \frac{1}{4}\int e^{2x}\sin x$

$\frac{5}{4}\int e^{2x}\sin x\,dx = \frac{1}{2}\sin x e^{2x} - \frac{1}{4}\cos x e^{2x}$

$= \frac{2}{5}\sin x e^{2x} - \frac{1}{5}\cos x e^{2x} + C$

$\boxed{\frac{1}{5}e^{2x}(2\sin x - \cos x) + C}$

If P(0) = 8, for what value of P is the population growing the fastest?

$$\frac{dP}{dt} = 52y - 12y^2$$

$12y(\frac{13}{3} - y)$

$\frac{\frac{13}{3}}{2} = \boxed{\frac{13}{6}}$

$$\int_0^1 \frac{1}{x^3} dx$$

$\lim_{b\to 0} \int_b^1 x^{-3}dx$

$-\frac{1}{2}x^{-2}\Big]_b^1$

$-\frac{1}{2}(1)^{-2} + \frac{1}{2}(b)^{-2}$

$\lim_{b\to 0}\left[-\frac{1}{2} + \frac{1}{2b^2}\right]$ $\quad$ $\boxed{\text{diverges}}$

$-\frac{1}{2} + \frac{1}{0}$ $\quad$ DNE