AP Calculus BC
Unit 8 – (Days 5 – 8) – QUIZ REVIEW

Name: Answer Key*

Evaluate the integral.

1) $\int \sin^{5}x\cos^{4}x\,dx$ (5 odd)

$\int \sin^4x\cos^4x\sin x\,dx$

$\int (1-\cos^2x)^2(\cos^4x)\sin x\,dx$

$\int (1-2\cos^2x+\cos^4x)(\cos^4x)\sin x\,dx$

$-1\int (\cos^4x-2\cos^6x+\cos^8x)(-1)(\sin x)\,dx$

$u=\cos x$, $du=-\sin x\,dx$

$-1\int u^4-2u^6+u^8\,du$

$-1\left[\frac{1}{5}u^5-2\left(\frac{1}{7}u^7\right)+\frac{1}{9}u^9\right]+C$

$-\frac{1}{5}u^5+\frac{2}{7}u^7-\frac{1}{9}x^9+C$

$\boxed{-\frac{1}{5}\cos^5x+\frac{2}{7}\cos^7x-\frac{1}{9}\cos^9x+C}$

2) $\int \frac{5}{x^3-x}\,dx$

$x^3-x=x(x^2-1)=x(x+1)(x-1)$

$\frac{5}{x^3-x}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$ (denominators $x(x+1)(x-1)$)

$5=A(x+1)(x-1)+B(x)(x-1)+C(x)(x+1)$

$x=-1$: $5=A(-1+1)(-1-1)+B(-1)(-1-1)+C(-1)(-1+1)$

$5=2B$ $\boxed{B=5/2}$

$x=0$: $5=A(0+1)(0-1)+B(0)(0-1)+C(0)(0+1)$

$5=-A$ $\boxed{A=-5}$

$x=1$: $5=A(1+1)(1-1)+B(1)(1+1)+C(1)(1+1)$

$5=2C$ $\boxed{C=5/2}$

$\int \frac{-5}{x}\,dx+\int \frac{5/2}{x+1}\,dx+\int \frac{5/2}{x-1}\,dx$

$\boxed{-5\ln|x|+\frac{5}{2}\ln|x+1|+\frac{5}{2}\ln|x-1|+C}$

3) $\int \sec^{2}x\tan x\,dx$ (2 even)

$\int \sec^2x(\tan x)(\sec^2x)\,dx$

$\int (\tan^2x+1)(\tan x)(\sec^2x)\,dx$

$\int (\tan^3x+\tan x)(\sec^2x)\,dx$

$u=\tan x$, $du=\sec^2x\,dx$

$\int u^3+u\,du$

$\frac{1}{4}u^4+\frac{1}{2}u^2+C$

$\boxed{\frac{1}{4}\tan^4x+\frac{1}{2}\tan^2x+C}$

4) $\int e^{2x}\sin x\,dx$

$u=\sin x$ $v=\frac{1}{2}e^{2x}$

$du=\cos x\,dx$ $dv=e^{2x}\,dx$

$(\sin x)(\frac{1}{2}e^{2x})-\int(\frac{1}{2}e^{2x})(\cos x)\,dx$

$u=\cos x$ $v=\frac{1}{4}e^{2x}$

$du=-\sin x\,dx$ $dv=\frac{1}{2}e^{2x}\,dx$

$-\left[(\cos x)(\frac{1}{4}e^{2x})-\int(\frac{1}{4}e^{2x})(-\sin x)\,dx\right]$

$-\frac{1}{4}e^{2x}\cos x+\int -\frac{1}{4}e^{2x}\sin x\,dx$

$-\frac{1}{4}\int e^{2x}\sin x\,dx$

$\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x-\frac{1}{4}\int e^{2x}\sin x\,dx$

$\frac{5}{4}\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x$

$\frac{1}{2}\cdot\frac{4}{5}=\frac{4}{10}$

$-\frac{1}{4}\cdot\frac{4}{5}$

$\boxed{\int e^{2x}\sin x\,dx=\frac{2}{5}e^{2x}\sin x-\frac{1}{5}e^{2x}\cos x+C}$

5) $\int \sin^{②\text{ even}} x\,dx$

$\int \frac{1}{2}(1-\cos 2x)\,dx$

$\int \frac{1}{2}dx - \int \frac{1}{2}\cos 2x\,dx$

$\frac{1}{2}x$ $\quad -\frac{1}{2}\int \cos 2x\,dx$

$-\frac{1}{2}\left[\frac{1}{2}\sin 2x\right]$

$-\frac{1}{4}\sin 2x$

$\boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

6) $\int \ln(5x)\,dx$

$u = \ln(5x) \qquad v = x$

$du = \frac{5}{5x}dx \qquad dv = 1\,dx$

$(\ln 5x)(x) - \int (x)\left(\frac{5}{5x}\right)dx$

$-\int dx$

$-x$

$\boxed{x\ln 5x - x + C}$

7) $\int \frac{2}{x^2+x}\,dx$

$x^2 + x = x(x+1)$

$\frac{2}{x^2+x} = \frac{A}{x}\,^{(x+1)} + \frac{B}{x+1}\,^{(x)}$

$2 = A(x+1) + B(x)$

$x=-1$: $\quad 2 = A(-1+1) + B(-1)$

$2 = -B \qquad B = -2$

$x=0$: $\quad 2 = A(0+1) + B(0)$

$2 = A$

$\int \frac{2}{x}\,dx + \int \frac{-2}{x+1}\,dx$

$2\ln|x| \quad -2\ln|x+1|$

$\boxed{2\ln|x| - 2\ln|x+1| + C}$

8) $\int \sec^3 x \tan^{③\text{ odd}} x\,dx$

$\int \sec^2 x \tan^2 x \sec x \tan x\,dx$

$\int (\sec^2 x)(\sec^2 x - 1)(\sec x \tan x)\,dx$

$\int (\sec^4 x - \sec^2 x)(\sec x \tan x)\,dx$

$u = \sec x$

$du = \sec x \tan x\,dx$

$\int u^4 - u^2\,du$

$\frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

$\boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$