AP Calculus BC

Unit 8 – Days 1 – 4 – QUIZ REVIEW

Name: Answer Key*

Directions: ***Match the problem in Column 1 with the solution in Column 2.***

Column 1 (Problems)		Column 2 (Answers)
1. $\frac{d}{dx}[\arcsin(36x)]$	g.	~~a.~~ $7\arcsin\left(\frac{x-2}{3}\right)+C$
2. $\int(\tan x)\ln(\cos x)\,dx$	l.	~~b.~~ $\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+C$
3. $\int x^2(x^3+5)^6dx$	p.	~~c.~~ $\frac{1}{3}\arctan\left(\frac{\sin x}{3}\right)+C$
4. $\int\frac{2}{x^2+2}dx$	b.	~~d.~~ $\frac{1}{\lvert x\rvert\sqrt{\frac{1}{4}x^2-1}}$
5. $\int\frac{e^x dx}{(e^x+2)^2}$	s.	~~e.~~ $3x-9\arctan\left(\frac{x}{3}\right)+C$
6. $\frac{d}{dx}\left[\operatorname{arcsec}\left(\frac{x}{2}\right)\right]$	d.	~~f.~~ $\frac{16x-4}{\lvert 4x-1\rvert}$
7. $\int x\sqrt{1-x}\,dx$	o.	~~g.~~ $\frac{6}{\sqrt{1-36x^2}}$
8. $\int\frac{7}{\sqrt{5+4x-x^2}}dx$	a.	~~h.~~ $\sqrt{2x+1}+C$
9. $\int\frac{3x^2}{x^2+9}dx$	e.	~~i.~~ $-2\sqrt{9-x^2}-3\arcsin\left(\frac{x}{3}\right)+C$
10. $\frac{d}{dx}[4^x]$	k.	~~j.~~ $\frac{1}{2}\operatorname{arcsec}\lvert 2x\rvert+C$
11. $\int\frac{5x}{\sqrt{x+2}}dx$	r.	~~k.~~ $4^x\ln 4$
12. $\int\frac{\cos x}{9+\sin^2 x}dx$	c.	~~l.~~ $-\frac{1}{2}[\ln(\cos x)]^2+C$
13. $\int\frac{2x-3}{\sqrt{9-x^2}}dx$	i.	~~m.~~ $\frac{1}{2\sqrt{x}(1+x)}$
14. $\frac{d}{dx}[\lvert 4x-1\rvert]$	f.	~~n.~~ $-5\csc(5x)\cot(5x)$
15. $\int\frac{1}{\sqrt{2x+1}}dx$	h.	~~o.~~ $\frac{2}{5}(1-x)^{\frac{5}{2}}-\frac{2}{3}(1-x)^{\frac{3}{2}}+C$
16. $\int\frac{1}{2x\sqrt{4x^2-1}}dx$	j.	~~p.~~ $\frac{1}{21}(x^3+5)^7+C$
17. $\frac{d}{dx}[\arctan\sqrt{x}]$	m.	~~q.~~ $\frac{1}{2}\ln(x^2+9)+\arctan\left(\frac{x}{3}\right)+C$
18. $\frac{d}{dx}[\csc(5x)]$	n.	~~r.~~ $\frac{10}{3}(x+2)^{3/2}-20\sqrt{x+2}+C$
19. $\int\sin^3 3x\cos 3x\,dx$	t.	~~s.~~ $\frac{-1}{e^x+2}+C$
20. $\int\frac{x+3}{x^2+9}dx$	q.	~~t.~~ $\frac{1}{12}\sin^4 3x+C$

① $\frac{d}{dx}[\arcsin(6x)]$

$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

$= \frac{6}{\sqrt{1-(6x)^2}}$

$= \boxed{\frac{6}{\sqrt{1-36x^2}}}$ $\boxed{g}$

② $-1\int (-\tan x) \ln(\cos x)\, dx$

$u = \ln(\cos x)$

$du = \frac{-\sin x}{\cos x} dx$

$= -\tan x\, dx$

$= -1 \int u\, du$

$= \frac{-1\, u^2}{2} + C$

$= \boxed{-\frac{1}{2}[\ln(\cos x)]^2 + C}$ $\boxed{l}$

③ $\frac{1}{3}\int 3x^2 (x^3+5)^6\, dx$

$u = x^3+5$

$du = 3x^2 dx$

$= \frac{1}{3}\int u^6\, du$

$= \frac{1}{3}\left(\frac{1}{7}u^7\right) + C$

$= \frac{1}{21}u^7 + C$

$= \boxed{\frac{1}{21}(x^3+5)^7 + C}$ $\boxed{p.}$

(4) $2\int \frac{\cancel{2}}{x^2+2}dx$

$u = x$

$du = dx$

$a = \sqrt{2}$

$= 2\int \frac{du}{u^2+a^2}$

$= 2\left(\frac{1}{\sqrt{2}}\right)\arctan\left(\frac{x}{\sqrt{2}}\right)+C$

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

$= \boxed{\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+C}$ $\boxed{b.}$

(5) $\int \frac{e^x dx}{(e^x+2)^2}$

$u = e^x + 2$

$du = e^x dx$

$\int \frac{du}{u^2}$

$= \int u^{-2} du$

$= \frac{u^{-1}}{-1} + C$

$= -1(e^x+2)^{-1} + C$

$= \boxed{\frac{-1}{e^x+2} + C}$ $\boxed{5.}$

⑥ $\frac{d}{dx}\left[\operatorname{arcsec}\left(\frac{x}{2}\right)\right]$

$\frac{1}{2}x$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$= \frac{\frac{1}{2}}{\left|\frac{1}{2}x\right|\sqrt{\left(\frac{1}{2}x\right)^2-1}} = \frac{\cancel{\frac{1}{2}}}{\cancel{\frac{1}{2}}|x|\sqrt{\frac{1}{4}x^2-1}} = \boxed{\frac{1}{|x|\sqrt{\frac{1}{4}x^2-1}}} \quad \boxed{d.}$$

⑦ $\int x\sqrt{1-x}\,dx$

$u = 1-x$

$du = -1\,dx$

$-du = dx$

$u-1 = -x$

$-u+1 = x$

$= \int (-u+1)\sqrt{u}\,(-du)$

$= \int -(-u+1)(u^{1/2})\,du$

$= \int (u-1)(u^{1/2})\,du$

$= \int u^{3/2} - u^{1/2}\,du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{3/2} + C}$ $\boxed{D.}$

⑧ $\int \frac{7}{\sqrt{5+4x-x^2}}\,dx$

$-x^2+4x+5$

$-(x^2-4x-5)$

$-\left[x^2-4x+\left(\frac{-4}{2}\right)^2-5-\left(\frac{-4}{2}\right)^2\right]$

$-\left[(x-2)^2-9\right]$

$-(x-2)^2+9$

$= 7\int \frac{\cancel{7}}{\sqrt{9-(x-2)^2}}\,dx$

$u=x-2$
$du=dx$
$a=3$

$= 7\int \frac{du}{\sqrt{a^2-u^2}}$

$= \boxed{7\arcsin\left(\frac{x-2}{3}\right)+C}$ a.

⑨ $\int \frac{3x^2}{x^2+9}\,dx = \int 3\,dx + \int \frac{-27}{x^2+9}\,dx$

$u=x$
$du=dx$
$a=3$

$3x$

$-27\int \frac{du}{u^2+a^2}$

$-27\left(\frac{1}{3}\right)\arctan\left(\frac{x}{3}\right)+C$

$$\begin{array}{r} 3 \\ x^2+0x+9 \enclose{longdiv}{3x^2+0x+0} \\ -\ (\cancel{3x^2}+\cancel{0x}+27) \\ \hline -27 \end{array}$$

$3 + \frac{-27}{x^2+9}$

$\boxed{3x-9\arctan\left(\frac{x}{3}\right)+C}$ e.

(10) $\frac{d}{dx}[4^x] =$

$\frac{d}{dx}[a^u] = (\ln a)(a^u)(u')$

$(\ln 4)(4^x)(1)$

$= \boxed{4^x \ln 4}$ $\boxed{K.}$

(11) $\int \frac{5x}{\sqrt{x+2}}\,dx$

$u = x+2$
$du = dx$
$u-2 = x$

$5\int \frac{(u-2)}{\sqrt{u}}\,du$

$5\int (u-2)(u^{-1/2})\,du$

$5\int u^{1/2} - 2u^{-1/2}\,du$

$5\left[\frac{2}{3}u^{3/2} - \frac{2u^{1/2}}{\frac{1}{2}}\right] + C$

$\frac{10}{3}u^{3/2} - 20u^{1/2} + C$

$\boxed{\frac{10}{3}(x+2)^{3/2} - 20(x+2)^{1/2} + C}$

or

$\boxed{\frac{10}{3}(x+2)^{3/2} - 20\sqrt{x+2} + C}$ $\boxed{r.}$

(12) $\int \frac{\cos x}{9+\sin^2 x}\,dx$

$u=\sin x$
$du=\cos x\,dx$
$a=3$

$\int \frac{du}{a^2+u^2}$

$\boxed{\frac{1}{3}\arctan\left(\frac{\sin x}{3}\right)+C}$ $\boxed{C.}$

(13) $\int \frac{2x-3}{\sqrt{9-x^2}}\,dx$

$u=9-x^2$
$du=-2x\,dx$

$-1\int \frac{-1\,2x}{\sqrt{9-x^2}}\,dx + -3\int \frac{\cancel{-3}}{\sqrt{9-x^2}}\,dx$

$u=x$
$du=dx$
$a=3$

$u=9-x^2$
$du=-2x\,dx$

$= -1\int \frac{du}{\sqrt{u}}$

$= -3\int \frac{du}{\sqrt{a^2-u^2}}$

$-3\arcsin\left(\frac{x}{3}\right)+C$

$= -1\int u^{-1/2}\,du$

$= -1\left(2u^{1/2}\right)+C$

$= -2u^{1/2}$

$= -2(9-x^2)^{1/2}$

$\boxed{-2\sqrt{9-x^2}-3\arcsin\left(\frac{x}{3}\right)+C}$ $\boxed{I}$

(14) $\frac{d}{dx}\left[\,|4x-1|\,\right]$ $\qquad\qquad$ $\frac{d}{dx}\left[|u|\right] = \frac{u}{|u|}\cdot u'$

$$= \frac{4x-1}{|4x-1|}(4) = \boxed{\frac{16x-4}{|4x-1|}} \quad \boxed{f.}$$

(15) $\frac{1}{2}\int \frac{(2)1}{\sqrt{2x+1}}\,dx$ $\qquad$ $u = 2x+1$

$du = 2dx$

$$\frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$= \frac{1}{2}\int u^{-1/2}\,du$$

$$= \frac{1}{2}\left[2u^{1/2}\right] + C$$

$$= u^{1/2} + C$$

$$= (2x+1)^{1/2} + C$$

$$= \boxed{\sqrt{2x+1} + C} \quad \boxed{h.}$$

(16) $\frac{1}{2}\int \frac{2x}{2x\sqrt{4x^2-1}}\,dx$

$u = 2x$
$du = 2dx$
$a = 1$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{2}\left(\frac{1}{1}\right) \text{arcsec} \frac{|2x|}{1} + C$

$\boxed{\frac{1}{2} \text{arcsec} |2x| + C}$ $\boxed{\text{j}}$

(17) $\frac{d}{dx}[\arctan \sqrt{x}]$

$x^{1/2}$

$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$

$= \frac{\frac{1}{2}x^{-1/2}}{1+(x^{1/2})^2} = \frac{\frac{1}{2}x^{-1/2}}{1+x} = \boxed{\frac{1}{2\sqrt{x}(1+x)}}$ $\boxed{\text{m.}}$

(18) $\frac{d}{dx}[\csc(5x)]$

$\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

$= \boxed{-5\csc 5x \cot 5x}$ $\boxed{\text{n.}}$

(19) $\int \sin^3 3x \cos 3x\,dx$

$\frac{1}{3}\int 3(\sin 3x)^3 \cos 3x\,dx$

$u = \sin 3x$
$du = 3\cos 3x\,dx$

$\frac{1}{3}\int u^3\,du$

$\frac{1}{3}\left[\frac{1}{4}u^4\right] + C$

$\frac{1}{12}u^4 + C$

$\frac{1}{12}(\sin 3x)^4 + C$

$\boxed{\frac{1}{12}\sin^4(3x) + C}$ $\boxed{E.}$

(20) $\int \frac{x+3}{x^2+9}\,dx$

$u = x^2+9$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{2x}{x^2+9}\,dx + 3\int \frac{\cancel{3}}{x^2+9}\,dx$

$u = x$
$du = dx$
$a = 3$

$u = x^2+9$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

$3\int \frac{du}{u^2+a^2}$

$3\left(\frac{1}{3}\right)\arctan\left(\frac{x}{3}\right)$

$\frac{1}{2}\ln|u|$

$\frac{1}{2}\ln(x^2+9)$

$\boxed{\frac{1}{2}\ln(x^2+9) + \arctan\left(\frac{x}{3}\right) + C}$ $\boxed{9.}$