AP Calculus BC
Unit 8 – Day 9 – Assignment

Name: Answer Key *

Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

1) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}}\,dx$ $\boxed{\text{converges}}$ $\boxed{4}$	2) $\displaystyle\int_3^4 \frac{1}{(x-3)^{3/2}}\,dx$ $\boxed{\text{diverges}}$
3) $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,dx$ $\boxed{\text{diverges}}$	4) $\displaystyle\int_0^\infty e^{-x}\,dx$ $\boxed{\text{converges}}$ $\boxed{1}$

5) $$\int_1^{\infty} \frac{1}{x^2} dx$$ $\boxed{\text{converges}}$ $\boxed{1}$	6) $$\int_1^{\infty} \frac{3}{\sqrt[3]{x}} dx$$ $\boxed{\text{diverges}}$
7) $$\int_{-\infty}^{0} xe^{-2x} dx$$ $\boxed{\text{diverges}}$	8) $$\int_{-\infty}^{\infty} \frac{2}{4+x^2} dx$$ $\boxed{\text{converges}}$ $\boxed{\pi}$

① $\int_{0}^{4} \frac{1}{\sqrt{x}}\,dx = \lim_{c\to 0^+} \int_c^4 x^{-1/2}\,dx$

$x \neq 0$

$\left[2x^{1/2}\right]_c^4$

$\left[2\sqrt{4} - 2\sqrt{c}\right]$

$\lim_{c\to 0^+} \left[4 - 2\sqrt{c}\right]$

$= 4 - 2\sqrt{0} = \boxed{4}$ $\boxed{\text{converges}}$

② $\int_{3}^{4} \frac{1}{(x-3)^{3/2}}\,dx = \lim_{c\to 3^+} \int_c^4 (x-3)^{-3/2}\,dx$

$u = x-3$
$du = dx$

$x \neq 3$

$\left[-2(x-3)^{-1/2}\right]_c^4$

$\left[\frac{-2}{\sqrt{x-3}}\right]_c^4$

$\left[\frac{-2}{\sqrt{4-3}} - \frac{-2}{\sqrt{c-3}}\right]$

$\lim_{c\to 3^+} \left[-2 + \frac{2}{\sqrt{c-3}}\right]$

$-2 + \cancel{\frac{2}{\sqrt{3-3}}}$

DNE

$\boxed{\begin{array}{c}\infty \\ \text{diverges}\end{array}}$

(3) $\int_0^2 \frac{1}{(x-1)^2}\,dx$

$x \neq 1$

$$\int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$

$$\lim_{c\to 1^-} \int_0^c (x-1)^{-2}\,dx + \lim_{c\to 1^+} \int_c^2 (x-1)^{-2}\,dx$$

$$\left[-(x-1)^{-1}\right]_0^c \qquad \left[\frac{-1}{x-1}\right]_c^2$$

$$\left[\frac{-1}{x-1}\right]_0^c \qquad \left[\frac{-1}{2-1} - \frac{-1}{c-1}\right]$$

$$\left[\frac{-1}{c-1} - \frac{-1}{0-1}\right] \qquad \lim_{c\to 1^+}\left[-1 + \frac{1}{c-1}\right]$$

$$\lim_{c\to 1^-}\left[\frac{-1}{c-1} - 1\right] \qquad -1 + \frac{1}{\cancel{1-1}}$$

$$\frac{-1}{1-1} - 1 \qquad \text{DNE}$$

DNE

$$\infty - 1 \quad + \quad -1 + \infty$$

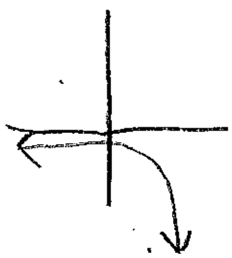
$\boxed{\infty}$ $\boxed{\text{diverges}}$

(4) $\int_0^{\infty} e^{-x}\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\,dx$

$$\left[-e^{-x}\right]_0^b$$

$$\left[-e^{-b} + e^{-0}\right]$$

$$\lim_{b\to\infty} \left[-e^{-b} + 1\right]$$



$$-e^{-\infty} + 1$$

$$0 + 1 = \boxed{1} \quad \boxed{\text{converges}}$$

(5) $\int_1^{\infty} \frac{1}{x^2}\,dx$

special type

$p = 2$

$$\frac{1}{2-1} = \boxed{1} \quad \boxed{\text{converges}}$$

(6) $\int_1^{\infty} \frac{3}{\sqrt[3]{x}}\,dx$

special type

$3\int_1^{\infty} \frac{1}{x^{1/3}}\,dx \quad p = 1/3 \quad \boxed{\infty} \quad \boxed{\text{diverges}}$

(7) $\int_{-\infty}^{0} x e^{-2x} dx = \lim_{b \to -\infty} \int_b^0 x e^{-2x} dx$

$u = x \qquad v = -\frac{1}{2}e^{-2x}$

$du = dx \qquad dv = e^{-2x}dx$

$(x)(-\frac{1}{2}e^{-2x}) - \int -\frac{1}{2}e^{-2x} dx$

$+\frac{1}{2}\int e^{-2x} dx$

$+\frac{1}{2}[-\frac{1}{2}e^{-2x}]$

$-\frac{1}{4}e^{-2x}$

$[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}]_b^0$

$[\cancel{-\frac{1}{2}(0)e^{-2(0)}} - \frac{1}{4}e^{-2(0)} + \frac{1}{2}(b)e^{-2b} + \frac{1}{4}e^{-2b}]$

$\lim_{b \to -\infty} [-\frac{1}{4} + \frac{1}{2}be^{-2b} + \frac{1}{4}e^{-2b}]$

$-\frac{1}{4} + \frac{1}{2}(-\infty)e^{-2(-\infty)} + \frac{1}{4}e^{-2(\infty)}$

$-\frac{1}{4} - \infty + 0 = \boxed{-\infty}$

$\boxed{\text{diverges}}$

8) $\int_{-\infty}^{\infty} \frac{2}{4+x^2}\,dx = \lim_{b\to-\infty} \int_b^0 \frac{2}{4+x^2}\,dx + \lim_{b\to\infty} \int_0^b \frac{2}{4+x^2}\,dx$

$u = x$
$du = dx$
$a = 2$

$2\int \frac{du}{a^2+u^2}$

$2\left(\frac{1}{2}\right)\arctan\left(\frac{x}{2}\right)$

$\left[\arctan\left(\frac{x}{2}\right)\right]_b^0$

$\left[\arctan\left(\frac{0}{2}\right) - \arctan\left(\frac{b}{2}\right)\right]$

$\left[\arctan 0 - \arctan\left(\frac{b}{2}\right)\right]$

$\lim_{b\to-\infty}\left[0 - \arctan\left(\frac{b}{2}\right)\right]$

$0 - \left(-\frac{\pi}{2}\right)$

same work

$\left[\arctan\left(\frac{x}{2}\right)\right]_0^b$

$\left[\arctan\left(\frac{b}{2}\right) - \arctan\left(\frac{0}{2}\right)\right]$

$\left[\arctan\left(\frac{b}{2}\right) - 0\right]$

$\lim_{b\to\infty}\left[\arctan\left(\frac{b}{2}\right) - 0\right]$

$\frac{\pi}{2} - 0$

$0 + \frac{\pi}{2} + \frac{\pi}{2} - 0 = \frac{2\pi}{2} = \boxed{\pi}$

$\boxed{\text{converges}}$