Day 9 Notes: Improper Integrals

Improper integrals occur when:
-one (or both) limit of integration is ∞
-there is a discontinuity in the integrand somewhere between the limits of integration

CONTINUOUS FUNCTIONS

1. **f** (x) is continuous on [a, ∞)⇒

$$\int_{a}^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_{a}^{b} f(x)\,dx$$

Example 1:

$$\int_{0}^{\infty} xe^{-x}\,dx$$

2. If f(x) is continuous on (-∞, b], then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_{a}^{b} f(x)\,dx$$

Example 2:

$$\int_{-\infty}^{0} \sin\frac{x}{2}\,dx$$

3. If f is continuous on (-∞, ∞), then

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

$$= \lim_{a \to -\infty} \int_{a}^{c} f(x)\,dx + \lim_{b \to \infty} \int_{c}^{b} f(x)\,dx$$

Example 3:

$$\int_{-\infty}^{\infty} \frac{e^x}{1 + e^{2x}}\,dx$$

Example 4:

$$\int_0^{\pi/2} tan x\, dx$$

> 2. If f is continuous on (a, b], then
> $$\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx$$

Example 5:

$$\int_0^2 \frac{1}{x^2}\,dx$$

3. If f is continuous on [a, b] except at x = c, where there is an infinite discontinuity, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \lim_{c \to d^-} \int_a^d f(x)\,dx + \lim_{c \to d^+} \int_d^b f(x)\,dx$$

Example 6:

$$\int_0^2 \frac{1}{(x-1)^2}\,dx$$

A special type of improper integral:

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{p-1}, & p > 1 \\ \text{diverges}, & p \le 1 \end{cases}$$

Example 7:

$$\int_1^\infty \frac{1}{x^3}\,dx$$

Example 8:

$$\int_1^\infty \frac{4}{\sqrt{x}}\,dx$$

Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

1) $$\int_0^4 \frac{1}{\sqrt{x}}\,dx$$	2) $$\int_3^4 \frac{1}{(x-3)^{3/2}}\,dx$$
3) $$\int_0^2 \frac{1}{(x-1)^2}\,dx$$	4) $$\int_0^\infty e^{-x}\,dx$$

5)

$$\int_1^\infty \frac{1}{x^2}\,dx$$

6)

$$\int_1^\infty \frac{3}{\sqrt[3]{x}}\,dx$$

7)

$$\int_{-\infty}^0 xe^{-2x}\,dx$$

8)

$$\int_{-\infty}^\infty \frac{2}{4+x^2}\,dx$$