AP Calculus BC
Unit 8 – Day 7 – Assignment

Name: Answer Key*

Evaluate the indefinite integral.

1)

$\frac{1}{3}\int 3\sec 3x\,dx$

$u = 3x$
$du = 3dx$

$\frac{1}{3}\int \sec u\,du$

$\frac{1}{3}\ln|\sec u + \tan u| + C$

$\boxed{\frac{1}{3}\ln|\sec 3x + \tan 3x| + C}$

2)

$\int \sec^2 x \tan x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$\int u\,du$

$\frac{1}{2}u^2 + C$

$\boxed{\frac{1}{2}\tan^2 x + C}$

3) Case #2

$\int \tan^{3} 2x \sec^3 2x\,dx$ odd

$\int (\tan^2 2x)(\sec^2 2x)(\sec 2x)(\tan 2x)\,dx$

$\int (\sec^2 2x - 1)(\sec^2 2x)(\sec 2x)(\tan 2x)\,dx$

$\frac{1}{2}\int (\sec^4 2x - \sec^2 2x)\,2(\sec 2x \tan 2x)\,dx$

$\frac{1}{2}\int u^4 - u^2\,du$

$u = \sec 2x$
$du = 2\sec 2x \tan 2x\,dx$

$\frac{1}{2}\left[\frac{1}{5}u^5 - \frac{1}{3}u^3\right] + C$

$\frac{1}{10}u^5 - \frac{1}{6}u^3 + C$

$\boxed{\frac{1}{10}\sec^5 2x - \frac{1}{6}\sec^3 2x + C}$

4) Case #1

$\int \sec^4 5x\,dx$

$\int (\sec^2 5x)(\sec^2 5x)\,dx$

$\int (1 + \tan^2 5x)(\sec^2 5x)\,dx$

$\int \sec^2 5x + \tan^2 5x \sec^2 5x\,dx$

$\frac{1}{5}\int 5\sec^2 5x\,dx + \frac{1}{5}\int 5\tan^2 5x \sec^2 5x\,dx$

$u = 5x$
$du = 5dx$

$\frac{1}{5}\int \sec^2 u\,du$

$\frac{1}{5}(\tan u)$

$\frac{1}{5}\tan(5x)$

$u = \tan 5x$
$du = 5\sec^2 5x\,dx$

$\frac{1}{5}\int u^2\,du$

$\frac{1}{5}\left(\frac{1}{3}u^3\right)$

$\frac{1}{15}\tan^3 5x$

$\boxed{\frac{1}{5}\tan 5x + \frac{1}{15}\tan^3 5x + C}$

Case #4

Case #5: Change to sine & cosine!

5)

odd

$\int (\sec^2 \pi x)(\sec \pi x)\,dx = \int \sec^3 \pi x\,dx$

$u = \sec \pi x \qquad v = \frac{1}{\pi}\tan \pi x$

$du = \pi \sec \pi x \tan \pi x \qquad dv = \sec^2 \pi x\,dx$

$(\sec \pi x)(\frac{1}{\pi}\tan \pi x) - \int (\frac{1}{\pi}\tan \pi x)(\pi \sec \pi x \tan \pi x)\,dx$

$-\int \tan^2 \pi x \sec \pi x\,dx$

$-\int (\sec^2 \pi x - 1)(\sec \pi x)\,dx$

$-\int \sec^3 \pi x - \sec \pi x\,dx$

$-\int \sec^3 \pi x + \int \sec \pi x\,dx$

$\frac{1}{\pi}\ln|\sec \pi x + \tan \pi x|$

$\int \sec^3 \pi x\,dx = \frac{1}{\pi}\sec \pi x \tan \pi x - \int \sec^3 \pi x\,dx + \frac{1}{\pi}\ln|\sec \pi x + \tan \pi x|$

$2\int \sec^3 \pi x\,dx = \frac{1}{\pi}\sec \pi x \tan \pi x + \frac{1}{\pi}\ln|\sec \pi x + \tan \pi x|$

$\boxed{\int \sec^3 \pi x\,dx = \frac{1}{2\pi}\sec \pi x \tan \pi x + \frac{1}{2\pi}\ln|\sec \pi x + \tan \pi x| + C}$

6)

$\int \frac{\tan^2 x}{\sec^5 x}\,dx$

$\int \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^5 x}}\,dx$

$= \int \frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \frac{\cos^{\cancel{5}\,3} x}{1}\,dx$

odd

$= \int \sin^2 x \cos^3 x\,dx$

$= \int (\sin^2 x)(\cos^2 x)(\cos x)\,dx$

$= \int (\sin^2 x)(1 - \sin^2 x)(\cos x)\,dx$

$\int (\sin^2 x - \sin^4 x)(\cos x)\,dx$

$u = \sin x$

$du = \cos x$

$\int u^2 - u^4\,du$

$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$\boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

7) $\int \tan^2 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^2\,du$

$\frac{1}{3}u^3 + C$

$\boxed{\frac{1}{3}\tan^3 x + C}$

Case #3

8) $\int \tan^2 x\,dx$

$\int (\sec^2 x - 1)\,dx$

$\int \sec^2 x\,dx - \int 1\,dx$

$\tan x \qquad -x$

$\boxed{\tan x - x + C}$