AP Calculus BC
Unit 8 – Day 6 – Assignment

Name: Answer Key*

Evaluate the indefinite integral.

1) $\int \cos^{3}x \sin^4 x\,dx$ (odd)

$\int (\cos^2 x)(\sin^4 x)(\cos x)\,dx$

$\int (1-\sin^2 x)(\sin^4 x)(\cos x)\,dx$

$\int (\sin^4 x - \sin^6 x)(\cos x)\,dx$

$u = \sin x$, $du = \cos x\,dx$

$\int u^4 - u^6\,du$

$\frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$\boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

2) $\frac{1}{2}\int \sin^5 2x \cos 2x\,dx$

$u = \sin 2x$, $du = 2\cos 2x\,dx$

$\frac{1}{2}\int u^5\,du$

$\frac{1}{2}\left[\frac{1}{6}u^6\right] + C$

$\boxed{\frac{1}{12}\sin^6 2x + 6}$

3) $\int \sin^{5}x \cos^2 x\,dx$ (odd)

$\int (\sin^4 x)(\cos^2 x)(\sin x)\,dx$

$\int (1-\cos^2 x)^2(\cos^2 x)(\sin x)\,dx$

$\int (1 - 2\cos^2 x + \cos^4 x)(\cos^2 x)(\sin x)\,dx$

$-\int (\cos^2 x - 2\cos^4 x + \cos^6 x)(\sin x)\,dx$

$u = \cos x$, $du = -\sin x\,dx$

$-\int u^2 - 2u^4 + u^6\,du$

$-\left[\frac{1}{3}u^3 - 2\left(\frac{1}{5}u^5\right) + \frac{1}{7}u^7\right] + C$

$-\frac{1}{3}u^3 + \frac{2}{5}u^5 - \frac{1}{7}u^7 + C$

$\boxed{-\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C}$

4) $\int \cos^{3}\left(\frac{x}{3}\right) dx$ (odd)

$\int \cos^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right) dx$

$\int \left(1 - \sin^2\left(\frac{x}{3}\right)\right)\cos\left(\frac{x}{3}\right) dx$

$\int \cos\left(\frac{x}{3}\right) - \sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right) dx$

$3\int \frac{1}{3}\cos\left(\frac{x}{3}\right) dx - 3\int \frac{1}{3}\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right) dx$

$u = \frac{1}{3}x$, $du = \frac{1}{3}dx$

$3\int \cos u\,du$

$3\sin u$

$\boxed{3\sin\left(\frac{1}{3}x\right)}$

$u = \sin\left(\frac{x}{3}\right)$, $du = \frac{1}{3}\cos\left(\frac{x}{3}\right)dx$

$-3\int u^2\,du$

$-3\left[\frac{1}{3}u^3\right] + C$

$-u^3 + C$

$\boxed{-\sin^3\left(\frac{x}{3}\right) + C}$

$\boxed{3\sin\left(\frac{x}{3}\right) - \sin^3\left(\frac{x}{3}\right) + C}$

5) $\int \frac{\sin^{⑤}x}{\sqrt{\cos x}}dx$ (odd)

$\int \sin^5 x (\cos x)^{-1/2} dx$

$\int (\sin^4 x)(\cos x)^{-1/2}(\sin x)\,dx$

$\int (1-\cos^2 x)^2 (\cos x)^{-1/2}(\sin x)\,dx$

$\int (1-2\cos^2 x+\cos^4 x)(\cos x)^{-1/2}(\sin x)\,dx$

$-\int [(\cos x)^{-1/2} - 2(\cos x)^{3/2} + (\cos x)^{7/2}](\sin x)\,dx$

$u = \cos x$
$du = -\sin x\,dx$

$-\int u^{-1/2} - 2u^{3/2} + u^{7/2}\,du$

$-\left[2u^{1/2} - 2\left(\frac{2}{5}u^{5/2}\right) + \frac{2}{9}u^{9/2}\right] + C$

$-2u^{1/2} + \frac{4}{5}u^{5/2} - \frac{2}{9}u^{9/2} + C$

$\boxed{-2(\cos x)^{1/2} + \frac{4}{5}(\cos x)^{5/2} - \frac{2}{9}(\cos x)^{9/2} + C}$

6) $\int \cos^{②}(3x)dx$ (even)

$\int \frac{1}{2}(1+\cos 6x)\,dx$

$\int \frac{1}{2} + \frac{1}{2}\cos 6x\,dx$

$\int \frac{1}{2}dx + \int \frac{1}{2}\cos 6x\,dx$

$\frac{1}{2}x$

$(\frac{1}{6})\frac{1}{2}\int 6\cos 6x\,dx$

$u = 6x$
$du = 6\,dx$

$\frac{1}{12}\int \cos u\,du$

$\frac{1}{12}\sin u$

$\frac{1}{12}\sin(6x)$

$\boxed{\frac{1}{2}x + \frac{1}{12}\sin(6x) + C}$

7) $\int x\sin^2 x\,dx$

$u = x$
$du = dx$

$v = \frac{1}{2}x - \frac{1}{4}\sin(2x)$
$dv = \sin^2 x\,dx$

$\int \sin^2 x\,dx$

$\int \frac{1}{2}(1-\cos 2x)\,dx$

$\int \frac{1}{2}dx - \int \frac{1}{2}\cos 2x\,dx$

$\frac{1}{2}x$

$-\frac{1}{2}(\frac{1}{2})\int \cos 2x\,dx$

$u = 2x$
$du = 2dx$

$-\frac{1}{4}\int \cos u\,du$

$-\frac{1}{4}\sin u$

$-\frac{1}{4}\sin(2x)$

$(x)(\frac{1}{2}x - \frac{1}{4}\sin(2x)) - \int \frac{1}{2}x - \frac{1}{4}\sin(2x)\,dx$

$-\int \frac{1}{2}x\,dx + \int \frac{1}{4}\sin(2x)\,dx$

$-\frac{1}{2}\int x\,dx$

$-\frac{1}{2}(\frac{1}{2}x^2)$

$-\frac{1}{4}x^2$

$\frac{1}{2}\frac{1}{4}\int 2\sin(2x)\,dx$

$u = 2x$
$du = 2dx$

$\frac{1}{8}\int \sin u\,du$

$\frac{1}{8}(-\cos u)$

$-\frac{1}{8}\cos(2x)$

$\frac{1}{2}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{4}x^2 - \frac{1}{8}\cos(2x) + C$

$\boxed{\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C}$