AP Calculus BC
Unit 8 – Integration Techniques

Day 5 Notes: Integration by Parts

We use integration by parts (IBP) for integrands involving products of **unrelated algebraic and transcendental functions.

$$\int u\,dv = uv - \int v\,du$$

-Here is an acronym that will be helpful when trying to decide which function in the integrand should be u:

LIATE

L = logarithm
I = inverse trig
A = algebraic
T = trig
E = exponential

(Whichever function comes first in the acronym should be defined as u.)

Example 1:

$$\int xe^{2x}dx$$

$u = x \quad v = \frac{1}{2}e^{2x}$

$du = dx \quad dv = \int e^{2x}dx$

$\frac{1}{2}\int e^{2x}dx$

$u = 2x$, $du = 2dx$: $\frac{1}{2}\int e^{u}du$

$(x)(\frac{1}{2}e^{2x}) - \frac{1}{2}\int \cancel{\frac{1}{2}}e^{2x}dx\,(2)$

$-\frac{1}{4}\int e^{u}du \quad u = 2x,\ du = 2dx$

$\boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C}$

Example 2:

$$\int 1\,\ln x\,dx$$

$u = \ln x \quad v = x$

$du = \frac{1}{x}dx \quad dv = \int 1\,dx$

$(\ln x)(x) - \int (x)\frac{1}{x}\,dx$

$-\int 1\,dx$

$\boxed{x\ln x - x + C}$

Example 3:

$$\int x^3\ln x\,dx$$

$u = \ln x \quad v = \frac{1}{4}x^4$

$du = \frac{1}{x}dx \quad dv = \int x^3dx$

$(\ln x)(\frac{1}{4}x^4) - \int (\frac{1}{4}x^4)(\frac{1}{x})dx$

$-\int \frac{1}{4}x^3dx$

$-\frac{\frac{1}{4}x^4}{4} + C$

$\boxed{\frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C}$

Example 4:

$$\int \theta\sec\theta\tan\theta\,d\theta$$

$u = \theta \quad v = \sec\theta$

$du = 1\,d\theta \quad dv = \int \sec\theta\tan\theta\,d\theta$

$(\theta)(\sec\theta) - \int (\sec\theta)\,d\theta$

$\boxed{\theta\sec\theta - \ln|\sec\theta + \tan\theta| + C}$

$\boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

Example 5:

$$\int x\sqrt{x-3}\,dx$$

Why is IBP not appropriate for this integrand?

* 2 Algebraic Functions *

$u = x - 3$
$du = dx$
$u + 3 = x$

$$\int (u+3)(\sqrt{u})\,du$$
$$\int (u+3)(u^{1/2})\,du$$
$$\int u^{3/2} + 3u^{1/2}\,du$$
$$\frac{2}{5}u^{5/2} + \frac{3u^{3/2}}{3/2} + C$$

$3 \cdot \frac{2}{3}$

$$\frac{2}{5}u^{5/2} + 2u^{3/2} + C$$
$$\boxed{\frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + C}$$

Example 6:

$$\int x^2e^{-x}\,dx$$

$u = x^2 \quad v = -e^{-x}$
$du = 2x\,dx \quad dv = \int e^{-x}\,dx$

$$(x^2)(-e^{-x}) - \int -e^{-x}(2x)\,dx$$
$$+ \int 2xe^{-x}\,dx$$

$u = 2x \quad v = -e^{-x}$
$du = 2dx \quad dv = \int e^{-x}\,dx$

$$+(2x)(-e^{-x}) - \int(-e^{-x})(2dx)$$
$$+2\int e^{-x}\,dx$$
$$-2e^{-x} + C$$

*Answer at top *

Example 7:

$$\int e^x \sin 2x\,dx$$

$u = \sin(2x) \quad v = e^x$
$du = 2\cos(2x)\,dx \quad dv = e^x\,dx$

$$(\sin 2x)(e^x) - \int (e^x)(2\cos 2x)\,dx$$
$$-2\int e^x \cos 2x\,dx$$

$u = \cos 2x \quad v = e^x$
$du = -2\sin 2x\,dx \quad dv = e^x\,dx$

$$-2\left[(\cos 2x)(e^x) - \int (e^x)(-2\sin 2x)\right]$$
$$2\cos 2x e^x - 4\boxed{\int e^x \sin 2x\,dx}$$

$$\int e^x \sin 2x\,dx = e^x \sin 2x + 2e^x\cos 2x - 4\int e^x \sin 2x\,dx$$
$$5\int e^x \sin 2x\,dx = e^x \sin 2x + 2e^x \cos 2x$$
$$\int e^x \sin 2x\,dx = \boxed{\frac{1}{5}e^x \sin 2x + \frac{2}{5}e^x\cos 2x + C}$$

Example 8:

$$\int \sec^3 3x\,dx = \int \sec^2 x \sec x\,dx$$

$u = \sec x \quad v = \tan x$
$du = \sec x \tan x\,dx \quad dv = \sec^2 x\,dx$

$$(\sec x \tan x) - \int (\tan x)(\sec x \tan x)\,dx$$
$$-\int \tan^2 x \sec x\,dx$$
$$-\int (\sec^2 x - 1)(\sec x)\,dx$$
$$-\int \sec^3 x - \sec x\,dx$$
$$-\int \sec^3 x + \int \sec x\,dx$$
$$+\ln|\sec x + \tan x|$$

$$\int \sec^3 3x = \sec x \tan x - \int \sec^3 x + \ln|\sec x + \tan x|$$
$$2\int \sec^3 3x = \sec x \tan x + \ln|\sec x + \tan x|$$
$$\int \sec^3 3x = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$$