AP Calculus BC **Name:** Answer Key*

Unit 8 – Day 5 – Assignment

Evaluate the indefinite integral.

1)

$$\int xe^{-2x}dx$$

$u = x$ $\quad v = -\frac{1}{2}e^{-2x}$

$du = dx$ $\quad dv = e^{-2x}dx$

$$uv - \int v\,du$$

$$(x)(-\tfrac{1}{2}e^{-2x}) - \int(-\tfrac{1}{2}e^{-2x})(dx)$$

$$+\tfrac{1}{2}\int e^{-2x}dx$$

$$+\tfrac{1}{2}\left[-\tfrac{1}{2}e^{-2x}\right] + C$$

$$\boxed{-\tfrac{1}{2}xe^{-2x} - \tfrac{1}{4}e^{-2x} + C}$$

2)

$$\int x\cos x\,dx$$

$u = x$ $\quad v = \sin x$

$du = dx$ $\quad dv = \cos x\,dx$

$$(x)(\sin x) - \int(\sin x)dx$$

$$-[-\cos x] + C$$

$$\boxed{x\sin x + \cos x + C}$$

3)

$$\int \ln(3x)dx$$

$u = \ln(3x)$ $\quad v = x$

$du = \frac{3}{3x}dx$ $\quad dv = 1dx$

$$(\ln 3x)(x) - \int (x)(\tfrac{3}{3x})dx$$

$$-\int dx$$

$$-x \quad + C$$

$$\boxed{x\ln 3x - x + C}$$

4)

$$\int \frac{2x}{e^x}dx = \int 2xe^{-x}dx$$

$u = 2x$ $\quad v = -e^{-x}$

$du = 2dx$ $\quad dv = e^{-x}dx$

$$(2x)(-e^{-x}) - \int(-e^{-x})(2dx)$$

$$+2\int e^{-x}dx$$

$$+2[-e^{-x}] + C$$

$$\boxed{-2xe^{-x} - 2e^{-x} + C}$$

5)

$$\int x^2\cos x\,dx$$

1st Time:

$u = x^2$ $\quad v = \sin x$

$du = 2x\,dx$ $\quad dv = \cos x\,dx$

$$(x^2)(\sin x) - \int(\sin x)(2x)dx$$

2nd Time:

$u = 2x$ $\quad v = -\cos x$

$du = 2dx$ $\quad dv = \sin x\,dx$

$$-\left[(2x)(-\cos x) - \int(-\cos x)(2dx)\right]$$

$$2x\cos x \quad +-2\int\cos x\,dx$$

$$-2[\sin x] + C$$

$$\boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$$

6)

$$\int x\csc x\cot x\,dx$$

$u = x$ $\quad v = -\csc x$

$du = dx$ $\quad dv = \csc x\cot x\,dx$

$$(x)(-\csc x) - \int -\csc x\,dx$$

$$+\int \csc x\,dx$$

$$-\ln|\csc x + \cot x| \quad + C$$

$$\boxed{-x\csc x - \ln|\csc x + \cot x| + C}$$

$$\boxed{x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C}$$

7)

$$\int x^3e^x\,dx$$

1st Time:

$u = x^3 \quad v = e^x$

$du = 3x^2dx \quad dv = e^xdx$

$$(x^3)(e^x) - \int (e^x)(3x^2)\,dx$$

2nd Time:

$u = 3x^2 \quad v = e^x$

$du = 6x\,dx \quad dv = e^xdx$

$$-\left[(3x^2)(e^x) - \int (e^x)(6x)\,dx\right]$$

$$-3x^2e^x + \int e^x\,6x\,dx$$

3rd Time:

$u = 6x \quad v = e^x$

$du = 6dx \quad dv = e^xdx$

$$(6x)(e^x) - \int (e^x)(6dx)$$

$$-6\int e^x\,dx$$

$$-6[e^x] + C$$

8)

$$\int e^{2x}\sin x\,dx$$

1st Time

$u = \sin x \quad v = \frac{1}{2}e^{2x}$

$du = \cos x\,dx \quad dv = e^{2x}dx$

$$(\sin x)(\tfrac{1}{2}e^{2x}) - \int (\tfrac{1}{2}e^{2x})(\cos x)\,dx$$

2nd Time

$u = \cos x \quad v = \frac{1}{4}e^{2x}$

$du = -\sin x\,dx \quad dv = \frac{1}{2}e^{2x}dx$

$$-\left[(\cos x)(\tfrac{1}{4}e^{2x}) - \int (\tfrac{1}{4}e^{2x})(-\sin x)\,dx\right]$$

$$-\tfrac{1}{4}e^{2x}\cos x \;+ -\int \tfrac{1}{4}e^{2x}\sin x\,dx$$

$$-\tfrac{1}{4}\int e^{2x}\sin x\,dx$$

$$\int e^{2x}\sin x\,dx = \tfrac{1}{2}e^{2x}\sin x - \tfrac{1}{4}e^{2x}\cos x - \tfrac{1}{4}\int e^{2x}\sin x\,dx$$

$$\tfrac{5}{4}\int e^{2x}\sin x\,dx = \tfrac{1}{2}e^{2x}\sin x - \tfrac{1}{4}e^{2x}\cos x$$

$$\int e^{2x}\sin x\,dx = \tfrac{4}{5}\left(\tfrac{1}{2}e^{2x}\sin x - \tfrac{1}{4}e^{2x}\cos x\right)$$

$$= \boxed{\tfrac{2}{5}e^{2x}\sin x - \tfrac{1}{5}e^{2x}\cos x}$$

9)

$$\int 4\arccos x\,dx$$

$u = \arccos x \quad v = 4x$

$du = \dfrac{-1}{\sqrt{1-x^2}}dx \quad dv = 4dx$

$$(\arccos x)(4x) - \int (4x)\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$-\tfrac{1}{2}\cdot 4\int \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$$-2\int \frac{du}{\sqrt{u}}$$

$$-2\int u^{-1/2}du$$

$$-2\left[2u^{1/2}\right]$$

$$-4(1-x^2)^{1/2} + c$$

$$\boxed{4x\arccos x - 4\sqrt{1-x^2} + C}$$

10)

$$\int e^x\cos 2x\,dx$$

$u = \cos(2x) \quad v = e^x$

$du = -2\sin(2x)dx \quad dv = e^xdx$

$$(\cos 2x)(e^x) - \int (e^x)(-2\sin 2x)\,dx$$

$u = -2\sin 2x \quad v = e^x$

$du = -4\cos 2x \quad dv = e^xdx$

$$-\left[(-2\sin 2x)(e^x) - \int (e^x)(-4\cos 2x)\,dx\right]$$

$$2e^x\sin 2x \;+ -4\int e^x\cos 2x\,dx$$

$$\int e^x\cos 2x\,dx = e^x\cos 2x + 2e^x\sin 2x - 4\int e^x\cos 2x\,dx$$

$$5\int e^x\cos 2x\,dx = e^x\cos 2x + 2e^x\sin 2x$$

$$\int e^x\cos 2x\,dx = \boxed{\tfrac{1}{5}e^x\cos 2x + \tfrac{2}{5}e^x\sin 2x + C}$$