AP Calculus BC
Unit 8 – Integration Techniques

Day 4 Notes: Mixed Integration

Example 1: $\int \frac{4}{x^2+9}\,dx$	**Example 2:** $\int \frac{4x}{x^2+9}\,dx$
Example 3: $\int \frac{4x^2}{x^2+9}\,dx$	**Example 4:** $\int_0^1 \frac{x+3}{\sqrt{4-x^2}}\,dx$
Example 5: $\int (cotx)[\ln(sinx)]dx$	**Example 6:** $\int \tan^2 2x dx$

Example 7:	**Example 8:**
$\int \frac{x^2}{\sqrt{16-x^6}}\,dx$	$\int \frac{\mathbf{1}}{\mathbf{1}+\ \boldsymbol{e^x}}\,\mathbf{dx}$

**Procedures for Fitting Integrands to Basic Rules:

1) **Expand (numerator).** ***Example:*** $(1 + e^x)^2 = 1 + 2e^x + e^{2x}$

2) **Separate numerator.** ***Example:*** $\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$

3) **Complete the square.** ***Example:*** $\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$

4) **Divide improper rational function.** ***Example:*** $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$

5) **Add and subtract terms in numerator.**
Example: $\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$

6) **Use trigonometric identities.** ***Example:*** $\cot^2 x = \csc^2 x - 1$

7) **Multiply and divide by Pythagorean conjugate.**
Example: $\frac{1}{1+sinx} = \left(\frac{1}{1+sinx}\right)\left(\frac{1-sinx}{1-sinx}\right) = \frac{1-sinx}{1-sin^2x} = \frac{1-sinx}{cos^2x} = sec^2x - \frac{sinx}{cos^2x}$

AP Calculus BC **Name: _____**
Unit 8 – Day 4 – Assignment

Evaluate the indefinite integral.

1) $\int \frac{5}{(x-4)^5}\,dx$	2) $\int \left[x + \frac{1}{(3x-1)^3}\right] dx$
3) $\int \frac{x+1}{\sqrt{x^2+2x-4}}\,dx$	4) $\int \frac{1}{4+(x-1)^2}\,dx$
5) $\int \frac{2x}{x-4}\,dx$	6) $\int \left(\frac{1}{3x-1} - \frac{1}{3x+1}\right) dx$

7) $\int csc\pi x cot\pi x dx$	8) $\int \frac{2}{e^{-x}+1} dx$
9) $\int (tanx)[\ln(cosx)]dx$	10) $\int \frac{1+cosx}{sinx} dx$
11) $\int \frac{2}{3(secx-1)} dx$	12) $\int \frac{3x+2}{x^2+9} dx$

13) $$\int \frac{\tan(\frac{2}{x})}{x^2}\,dx$$

14) $$\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx$$

Solve the differential equation.

15) $$\frac{dr}{dt} = \frac{(1+\ e^t)^2}{e^t}$$

16) $$y' = \frac{1}{x\sqrt{4x^2-1}}$$