**AP Calculus BC** Name: Answer Key*
**Unit 8 – Day 4 – Assignment**

**Evaluate the indefinite integral.**

1)
$$5\int \frac{\cancel{5}}{(x-4)^5}\,dx$$

$u = x-4$
$du = dx$

$5\int \frac{du}{u^5}$

$5\int u^{-5}\,du$

$\frac{5u^{-4}}{-4} + C$

$-\frac{5}{4}(x-4)^{-4} + C = \boxed{\frac{-5}{4(x-4)^4} + C}$

2)
$$\int \left[x + \frac{1}{(3x-1)^3}\right] dx$$

$u = 3x-1$
$du = 3dx$

$\int x\,dx + \frac{1}{3}\int \frac{3\cancel{1}}{(3x-1)^3}\,dx$

$\frac{1}{2}x^2 + \frac{1}{3}\int \frac{du}{u^3}$

$\frac{1}{3}\int u^{-3}\,du$

$\frac{\frac{1}{3}u^{-2}}{-2} + C$

$\frac{1}{3}\cdot\frac{1}{2}$

$\frac{1}{2}x^2 - \frac{1}{6}(3x-1)^{-2} + C = \boxed{\frac{1}{2}x^2 - \frac{1}{6(3x-1)^2} + C}$

3)
$$\frac{1}{2}\int \frac{2(x+1)}{\sqrt{x^2+2x-4}}\,dx$$

$u = x^2+2x-4$
$du = 2x+2\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$\frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}\cdot\frac{2}{1}$

$\frac{\frac{1}{2}u^{1/2}}{1/2} + C$

$\boxed{(x^2+2x-4)^{1/2} + C}$

4)
$$\int \frac{1}{4+(x-1)^2}\,dx$$

$u = x-1$
$du = dx$
$a = 2$

$\int \frac{du}{a^2+u^2}$

$\boxed{\frac{1}{2}\arctan\left(\frac{x-1}{2}\right) + C}$

5)
$$\int \frac{2x}{x-4}\,dx$$

~~$u = x-4$~~
~~$du = dx$~~

$$\begin{array}{r} 2 \\ x-4\,\overline{)\,2x+0} \\ -(2x-8) \\ \hline 8 \end{array}$$

$\int 2 + \frac{8}{x-4}\,dx$

$\int 2\,dx + 8\int \frac{\cancel{8}}{x-4}\,dx$

$u = x-4$
$du = dx$

$2x + 8\int \frac{du}{u}$

$2x + 8\ln|u| + C$

$\boxed{2x + 8\ln|x-4| + C}$

6)
$$\int \left(\frac{1}{3x-1} - \frac{1}{3x+1}\right) dx$$

$\frac{1}{3}\int \frac{3\cancel{1}}{3x-1}\,dx - \frac{1}{3}\int \frac{3\cancel{1}}{3x+1}\,dx$

$u = 3x-1$
$du = 3dx$

$u = 3x+1$
$du = 3dx$

$\frac{1}{3}\int \frac{du}{u} - \frac{1}{3}\int \frac{u}{du}$

$\frac{1}{3}\ln|u| - \frac{1}{3}\ln|u|$

$\boxed{\frac{1}{3}\ln|3x-1| - \frac{1}{3}\ln|3x+1| + C}$

| | |
|---|---|
| 7) $\frac{1}{\pi}\int \pi \csc\pi x \cot\pi x \, dx$<br>$u=\pi x$<br>$du=\pi dx$<br>$\frac{1}{\pi}\int \csc u \cot u \, du$<br>$\frac{1}{\pi}(-\csc u)+C$<br>$\boxed{-\frac{1}{\pi}\csc(\pi x)+C}$ | 8) $\int \frac{2}{e^{-x}+1}dx \left(\frac{e^x}{e^x}\right)$<br>$2\int \frac{\cancel{2}e^x \, dx}{1+e^x}$<br>$u=1+e^x$<br>$du=e^x dx$<br>$2\int \frac{du}{u}$<br>$2\ln\|u\|+C$<br>$\boxed{2\ln(1+e^x)+C}$ |
| 9) $-1\int (-1)(\tan x)[\ln(\cos x)]dx$<br>$u=\ln(\cos x)$<br>$du=\frac{-\sin x}{\cos x}dx$<br>$=-\tan x dx$<br>$-1\int u \, du$<br>$-1\frac{u^2}{2}+C$<br>$\boxed{\frac{-1}{2}[\ln(\cos x)]^2+C}$ | 10) $\int \frac{1+\cos x}{\sin x}dx$<br>$\int \frac{1}{\sin x}dx+\int\frac{\cos x}{\sin x}dx$<br>$\int \csc dx + \int \cot x dx$<br>$\boxed{-\ln\|\csc x+\cot x\|+\ln\|\sin x\|+C}$ |
| 11) $\int \frac{2}{3(\sec x-1)}dx$<br>$\frac{2}{3}\int \frac{1}{\sec x-1}dx \frac{(\sec x+1)}{(\sec x+1)}$<br>$\frac{2}{3}\int \frac{\sec x+1}{\sec^2 x-1}dx$<br>$=\frac{2}{3}\int \frac{\sec x+1}{\tan^2 x}dx$<br>$\frac{2}{3}\int \frac{\sec x \, dx}{\tan^2 x}+\frac{2}{3}\int\frac{1}{\tan^2 x}dx$<br>$\frac{1}{\cancel{\cos x}} \quad \frac{\cos^2 x}{\sin^2 x}$<br>$\frac{2}{3}\int \frac{\cos x dx}{\sin^2 x}+\frac{2}{3}\int \cot^2 x dx$<br>$\frac{2}{3}\int\frac{du}{u^2}+\frac{2}{3}\int \csc^2 x-1 \, dx$<br>$u=\sin x$<br>$du=\cos x dx$ | 12) $\int \frac{3x+2}{x^2+9}dx$<br>$\frac{1}{2}\cdot 3\int \frac{\cancel{3}x}{x^2+9}dx + 2\int \frac{\cancel{2}}{x^2+9}dx$<br>$u=x^2+9$<br>$du=2xdx$<br>$\frac{3}{2}\int\frac{du}{u}+2\int\frac{du}{u^2+a^2}$<br>$u=x$<br>$du=dx$<br>$a=3$<br>$\frac{3}{2}\ln\|x^2+9\|+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+C$<br>$\boxed{\frac{3}{2}\ln(x^2+9)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+C}$ |

$\frac{2}{3}\int u^{-2}du \quad + \frac{2}{3}\int \csc^2 x + \frac{2}{3}\int 1 dx$

$\frac{2}{3}\frac{u^{-1}}{-1} \quad + \frac{2}{3}(\cot x) - \frac{2}{3}x$

$-\frac{2}{3}(\sin x)^{-1}$

$\boxed{-\frac{2}{3}[\csc x+\cot x+x]+C}$

13)

$$-\frac{1}{2}\int \frac{\tan\left(\frac{2}{x}\right)}{x^2}\,dx$$

$u = 2x^{-1}$
$du = -2x^{-2}dx$

$$-\frac{1}{2}\int \tan(u)\,du$$

$$(-1)\cdot -\frac{1}{2}\int \frac{-1\sin(u)}{\cos(u)}\,du$$

$$\frac{1}{2}\int \frac{du}{u}$$

$u = \cos u$
$du = -\sin u\,du$

$$\frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|\cos u| + C$$

$$\boxed{\frac{1}{2}\ln\left|\cos\left(\frac{2}{x}\right)\right| + C}$$

14)

$$\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx$$

$4x^2 - 8x + 3$
$4\left(x^2 - 2x + \frac{3}{4}\right)$
$4\left[x^2 - 2x + \left(\frac{-2}{2}\right)^2 + \frac{3}{4} - \left(\frac{-2}{2}\right)^2\right]$
$4\left[(x-1)^2 + -\frac{1}{4}\right]$
$4(x-1)^2 - 1$

$$\frac{1}{2}\int \frac{2 \cdot 1\,dx}{(x-1)\sqrt{4(x-1)^2-1}}$$

$u = 2(x-1) = 2x - 2$
$du = 2dx$
$a = 1$

$$= \frac{1}{2}\int \frac{du}{(x-1)\sqrt{u^2-a^2}}$$

$$= \int \frac{du}{2(x-1)\sqrt{u^2-a^2}}$$

$$= \int \frac{du}{u\sqrt{u^2-a^2}}$$

$$= \boxed{\operatorname{arcsec}|2(x-1)| + C}$$

**Solve the differential equation.**

15)

$$\frac{dr}{dt} = \frac{(1+e^t)^2}{e^t}$$

$$\int dr = \int \frac{(1+e^t)^2}{e^t}\,dt$$

~~$u = 1 + e^t$~~
~~$du = e^t dt$~~

$$r = \int \frac{1 + 2e^t + e^{2t}}{e^t}$$

$$r = \int (e^{-t} + 2 + e^t)\,dt$$

$$\boxed{r = -e^{-t} + 2t + e^t + C}$$

16)

$$y' = \frac{2 \cdot \frac{1}{2}}{x\sqrt{4x^2-1}}$$

$$\int dy = \int \frac{2 \cdot \frac{1}{2}}{x\sqrt{4x^2-1}}\,dx$$

$u = 2x$
$du = 2dx$
$a = 1$

$$y = \frac{1}{2}\int \frac{du}{x\sqrt{u^2-a^2}}$$

$$y = \int \frac{du}{2x\sqrt{u^2-a^2}}$$

$$y = \int \frac{du}{u\sqrt{u^2-a^2}}$$

$$y = \frac{1}{1}\operatorname{arcsec}\frac{|2x|}{1} + C$$

$$\boxed{y = \operatorname{arcsec}|2x| + C}$$