AP Calculus BC
Unit 8 – Day 3 – Assignment

Name: Answer Key*

Find or evaluate the integral.

1) $\frac{1}{2}\cdot 3\int \frac{2}{\sqrt{1-4x^2}}dx$

$u=2x$, $du=2dx$, $a=1$

$\frac{3}{2}\int \frac{du}{\sqrt{a^2-u^2}}$

$\frac{3}{2}\arcsin\frac{2x}{1}+C$

$\boxed{\frac{3}{2}\arcsin(2x)+C}$

2) $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

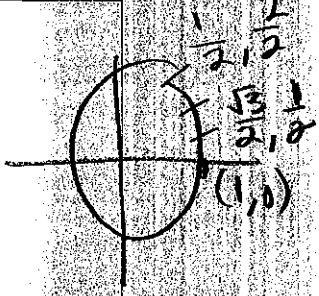
$u=x$, $du=dx$, $a=2$

$\int \frac{du}{\sqrt{a^2-u^2}}$

$\arcsin\left(\frac{x}{2}\right)\Big]_0^1$

$\arcsin\left(\frac{1}{2}\right)-\arcsin(0)$

$\pi/6-0=\boxed{\pi/6}$



3) $\frac{1}{3}\cdot 4\int \frac{3}{1+9x^2}dx$

$u=3x$, $du=3dx$, $a=1$

$\frac{4}{3}\int \frac{du}{a^2+u^2}$

$\frac{4}{3}\left(\frac{1}{1}\right)\arctan\left(\frac{3x}{1}\right)+C$

$\boxed{\frac{4}{3}\arctan(3x)+C}$

4) $\int \frac{1}{4+(x-1)^2}dx$

$u=x-1$, $du=dx$, $a=2$

$\int \frac{du}{a^2+u^2}$

$\boxed{\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)+C}$

5) $\frac{1}{2}\int \frac{2x}{x^4+16}dx$

$u=x^2$, $du=2xdx$, $a=4$

$\frac{1}{2}\int \frac{du}{a^2+u^2}$

$\frac{1}{2}\left(\frac{1}{4}\right)\arctan\left(\frac{x^2}{4}\right)+C$

$\boxed{\frac{1}{8}\arctan\left(\frac{x^2}{4}\right)+C}$

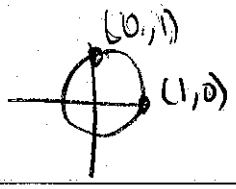
6) $\frac{1}{2}\int \frac{2x}{x\sqrt{x^4-4}}dx$

$u=x^2$, $du=2xdx$, $a=2$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{2}\left(\frac{1}{2}\right)\operatorname{arcsec}\frac{x^2}{2}+C$

$\boxed{\frac{1}{4}\operatorname{arcsec}\left(\frac{x^2}{2}\right)+C}$



7)

$$\frac{1}{2}\int_{-\sqrt{3}}^{0} \frac{2x}{1+x^2}\,dx$$

$u = x$, $du = dx$, $a = 1$

$u = 1 + x^2$, $du = 2x\,dx$

$$\frac{1}{2}\int \frac{du}{u}$$

$$\frac{1}{2}\ln|u|$$

$$\frac{1}{2}\ln|1+x^2|\Big]_{-\sqrt{3}}^{0}$$

$$\frac{1}{2}\ln|1+0^2| - \frac{1}{2}\ln|1+(-\sqrt{3})^2|$$

$$\frac{1}{2}\ln 1 - \frac{1}{2}\ln 4 = 0 - \frac{1}{2}\ln 4$$

$= -\frac{1}{2}\ln 4$ or $= \ln 4^{-1/2}$ or $= -\ln 2$

8)

$$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$$

$u = \sin x$, $du = \cos x\,dx$

$$\int \frac{du}{a^2+u^2}$$

$a = 1$

$$\arctan(\sin x)\Big]_0^{\pi/2}$$

$$\arctan(\sin\tfrac{\pi}{2}) - \arctan(\sin 0)$$

$$\arctan(1) - \arctan(0)$$

$$\pi/4 - 0 = \boxed{\pi/4}$$

9)

$$\int_{-2}^{2} \frac{dx}{x^2+4x+13}$$

$x^2+4x+\left(\frac{4}{2}\right)^2+13-\left(\frac{4}{2}\right)^2$

$(x+2)^2+9$

$$\int_{-2}^{2} \frac{dx}{(x+2)^2+9}$$

$u = x+2$, $du = dx$, $a = 3$

$$\int \frac{du}{u^2+a^2}$$

$$\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\Big]_{-2}^{2}$$

$$\frac{1}{3}\arctan(4/3) - \frac{1}{3}\arctan(0)$$

$$\frac{1}{3}\arctan(4/3) - 0 = \boxed{\frac{1}{3}\arctan\left(\frac{4}{3}\right)}$$

10)

$$\int \frac{2x+6-6}{x^2+6x+13}\,dx$$

$u = x^2+6x+13$, $du = 2x+6\,dx$

$$\int \frac{2x+6\,dx}{x^2+6x+13} + \int \frac{-6}{x^2+6x+13}\,dx$$

$$\int \frac{du}{u}$$

$$\ln|u|$$

$$x^2+6x+\left(\frac{6}{2}\right)^2+13-\left(\frac{6}{2}\right)^2$$

$$(x+3)^2+4$$

$$\int \frac{-6}{(x+3)^2+4}$$

$u = x+3$, $du = dx$, $a = 2$

$$-6\int \frac{du}{u^2+a^2}$$

$$\ln|x^2+6x+13| - 6\left(\frac{1}{2}\right)\arctan\left(\frac{x+3}{2}\right)+C$$

$$\boxed{\ln|x^2+6x+13| - 3\arctan\left(\frac{x+3}{2}\right)+C}$$

11)

$$\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$$

$x^2-2x+\left(\frac{-2}{2}\right)^2-\left(\frac{-2}{2}\right)^2$

$(x-1)^2-1$

$$\int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}$$

$u = x-1$, $du = dx$, $a = 1$

$$\int \frac{1}{u\sqrt{u^2-a^2}}$$

$$\frac{1}{1}\operatorname{arcsec}\frac{|x-1|}{1}+C$$

$$\boxed{\operatorname{arcsec}|x-1|+C}$$

12)

$$\int \frac{x}{x^4+2x^2+2}\,dx$$

$x^4+2x^2+\left(\frac{2}{2}\right)^2+2-\left(\frac{2}{2}\right)^2$

$(x^2+1)^2+1$

$$\frac{1}{2}\int \frac{2x}{(x^2+1)^2+1}\,dx$$

$u = x^2+1$, $du = 2x\,dx$, $a = 1$

$$\frac{1}{2}\int \frac{du}{u^2+a^2}$$

$$\boxed{\frac{1}{2}\arctan(x^2+1)+C}$$