

AP Calculus BC
Unit 8 – Integration Techniques

Day 2 Notes: Inverse Trig Functions - Differentiation

$$\frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Example 1: $\frac{d}{dx} [\arcsin(2x)]$

$$\frac{2}{\sqrt{1-(2x)^2}} = \boxed{\frac{2}{\sqrt{1-4x^2}}}$$

Example 2: $\frac{d}{dx} [\arctan(3x)]$

$$\frac{3}{1+(3x)^2} = \boxed{\frac{3}{1+9x^2}}$$

Example 3: $\frac{d}{dx} [\arcsin(\sqrt{x})] x^{1/2}$

$$\frac{\frac{1}{2}x^{-1/2}}{\sqrt{1-(\sqrt{x})^2}} = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \boxed{\frac{1}{2\sqrt{x-x^2}}}$$

Example 4: $\frac{d}{dx} [\operatorname{arcsec}(e^{2x})]$

$$\frac{2e^{2x}}{|e^{2x}|\sqrt{(e^{2x})^2-1}} = \frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}} = \boxed{\frac{2}{\sqrt{e^{4x}-1}}}$$

Example 5:

Differentiate $y = \arcsin(x) + \sqrt{x\sqrt{1-x^2}} \times (1-x^2)^{1/2}$

$$\frac{1}{\sqrt{1-x^2}} + (1)(1-x^2)^{1/2} + (x) \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$\frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{-x^2}{\sqrt{1-x^2}} = \frac{1-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}$$

$$= \frac{(1-x^2)^1}{(1-x^2)^{1/2}} + \sqrt{1-x^2}$$

$$= \sqrt{1-x^2} + \sqrt{1-x^2} = \boxed{2\sqrt{1-x^2}}$$