AP Calculus BC
Unit 8 – Integration Techniques

Day 1 Notes: Integration by Substitution

Example 1: $\int (x^2+1)^2(2x)\,dx$

$u = x^2+1$
$du = 2x\,dx$

$\int u^2\,du$

$\frac{1}{3}u^3 + C$

$\boxed{\frac{1}{3}(x^2+1)+C}$

Example 2: $\int 5\cos 5x\,dx$

$u = 5x$
$du = 5\,dx$

$\int \cos u\,du$

$\sin u + C$

$\boxed{\sin 5x + C}$

Example 3: $\frac{1}{2}\int (x^2+1)^2(2x)\,dx$

$u = x^2+1$
$du = 2x\,dx$

$\frac{1}{2}\int u^2\,du$

$\frac{1}{2}u^2 + C$

$\boxed{\frac{1}{2}(x^2+1)^2 + C}$

Example 4: $\int \sqrt{2x-1}\,dx$

$u = 2x-1$
$du = 2\,dx$

$\frac{1}{2}\int (2x-1)^{1/2}\,dx$

$\frac{1}{2}\int u^{1/2}\,du$

$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right] + C$

$\boxed{\frac{1}{3}(2x-1)^{3/2} + C}$

Example 5: $\int x\sqrt{2x-1}\,dx$

$u = 2x-1$
$du = 2\,dx$
$\frac{1}{2}du = dx$
$\frac{u+1}{2} = x$

$\int x(2x-1)^{1/2}\,dx$

$\int \left(\frac{u+1}{2}\right)(u)^{1/2}\left(\frac{1}{2}du\right)$

$\frac{1}{4}\int (u+1)(u^{1/2})\,du$

$\frac{1}{4}\int (u^{3/2} + u^{1/2})\,du$

$\frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$

$\frac{1}{10}u^{5/2} + \frac{1}{6}u^{3/2} + C$

$\boxed{\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C}$

Example 6: $\int \sin^2 3x\cos 3x\,dx$

$u = \sin 3x$
$du = 3\cos 3x\,dx$

$\frac{1}{3}\int [\sin(3x)]^2(\cos 3x)$

$\frac{1}{3}\int u^2\,du$

$\frac{1}{3}\left[\frac{1}{3}u^3\right] + C$

$\frac{1}{9}u^3 + C$

$\boxed{\frac{1}{9}\sin^3(3x) + C}$

1^2+1 / 0^2+1 $2(5)-1$ $2(1)-1$

Example 7: $\frac{1}{2}\int_0^1 (2x)(x^2+1)^3 dx$	**Example 8:** $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$
$u = x^2+1$, $du = 2x\,dx$	$u = 2x-1$, $du = 2dx$, $\frac{1}{2}du = dx$, $\frac{u+1}{2} = x$
$\frac{1}{2}\int_1^2 u^3 du$	$\int_1^5 x(2x-1)^{-1/2} dx$
$\frac{1}{2}\left[\frac{1}{4}u^4\right]_1^2$	$\int_1^9 \left(\frac{u+1}{2}\right)(u)^{-1/2}\left(\frac{1}{2}du\right)$
$\frac{1}{8}u^4\Big]_1^2$	$\frac{1}{4}\int_1^9 (u+1)(u)^{-1/2} du$
$\frac{1}{8}(2)^4 - \frac{1}{8}(1)^4$	$\frac{1}{4}\int_1^9 u^{1/2} + u^{-1/2} du$
$\frac{1}{8}(16) - \frac{1}{8}$	$\frac{1}{4}\left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_1^9$
$\frac{16}{8} - \frac{1}{8} = \boxed{\frac{15}{8}}$	$\frac{1}{6}u^{3/2} + \frac{1}{2}u^{1/2}\Big]_1^9$
	$\frac{1}{6}(9)^{3/2} + \frac{1}{2}(9)^{1/2} - \frac{1}{6}(1)^{3/2} - \frac{1}{2}(1)^{1/2}$
	$\frac{9}{2} + \frac{3}{2} - \frac{1}{6} - \frac{1}{2} = \boxed{\frac{16}{3}}$

Your Turn:

1) $\int(5x^2+1)^2(10x)dx$	2) $\int u^2\sqrt{u^3+2}du$
$u = 5x^2+1$, $du = 10x\,dx$	$x = u^3+2$, $dx = 3u^2du$
$\int u^2 du$	$\frac{1}{3}\int (3u^2)(u^3+2)^{1/2}(du)$
$\frac{1}{3}u^3 + C$	$\frac{1}{3}\int x^{1/2}dx$
$\boxed{\frac{1}{3}(5x^2+1)^3 + C}$	$\frac{1}{3}\left[\frac{2}{3}x^{3/2}\right] + C$ $\boxed{\frac{2}{9}(u^3+2)^{3/2} + C}$

3) $\int x\sqrt{x+2}\,dx$	4) $\int_0^1 x\sqrt{2x+1}dx$
$u = x+2$, $du = dx$, $x = u-2$	$u = 2x+1$, $du = 2dx$, $\frac{1}{2}du = dx$, $\frac{u-1}{2} = x$
$\int x(x+2)^{1/2}dx$	$\int_0^1 x(2x+1)^{1/2}dx$
$\int(u-2)(u)^{1/2}du$	$\int_1^3 \left(\frac{u-1}{2}\right)(u)^{1/2}\left(\frac{1}{2}du\right)$
$\int(u^{3/2} - 2u^{1/2})du$	$\frac{1}{4}\int_1^3 (u-1)(u^{1/2})du$
$\frac{2}{5}u^{5/2} - \frac{2u^{3/2}}{3/2} + C$ $2\cdot\frac{2}{3}$	$\frac{1}{4}\int_1^3 (u^{3/2} - u^{1/2})du$
$\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$	$\frac{1}{4}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^3$
$\boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$	$\frac{1}{10}u^{5/2} - \frac{1}{6}u^{3/2}\Big]_1^3 =$
	$\frac{1}{10}(3)^{5/2} - \frac{1}{6}(3)^{3/2} - \frac{1}{10}(1)^{5/2} + \frac{1}{6}(1)^{3/2}$
	$1.559 - 0.866 - 0.100 + 0.167 = \boxed{0.760}$