AP Calculus BC
Unit 8 – Day 1 – Assignment

Name: Answer Key*

#'s 1 – 10: Find the indefinite integral.

1) $\int (1+2x)^4 (2)(dx)$

$u = 1+2x$
$du = 2dx$

$\int u^4 du$

$\frac{1}{5}u^5 + C$

$\boxed{\frac{1}{5}(1+2x)^5 + C}$

2) $\frac{1}{2}\int (2)\sin 2x\,(dx)$

$u = 2x$
$du = 2dx$

$\frac{1}{2}\int \sin u\, du$

$\frac{1}{2}(-\cos u) + C$

$\boxed{-\frac{1}{2}\cos(2x) + C}$

3) $\int \frac{x}{\sqrt{1-x^2}}dx$

$u = 1-x^2$
$du = -2x$

$-\frac{1}{2}\int (-2x)(1-x^2)^{-1/2}(dx)$

$-\frac{1}{2}\int u^{-1/2}\, du$

$-\frac{1}{2}\left[2u^{1/2}\right] + C$

$-u^{1/2} + C$

$\boxed{-(1-x^2)^{1/2} + C}$

4) $\int \frac{1}{x^2}\cos\frac{1}{x}dx$

$u = x^{-1}$
$du = -x^{-2}dx$

$-1\int (-1x^{-2})\cos(x^{-1})(dx)$

$-1\int \cos(u)\, du$

$-1[\sin(u)] + C$

$\boxed{-\sin\left(\frac{1}{x}\right) + C}$

5) $\int \left(1+\frac{1}{x}\right)^3 \left(\frac{1}{x^2}\right)dx$

$u = 1+x^{-1}$
$du = -1x^{-2}dx$

$-1\int (-1)(1+x^{-1})^3 (x^{-2})\,dx$

$-1\int u^3\, du$

$-1\left(\frac{1}{4}u^4\right) + C$

$\boxed{-\frac{1}{4}\left(1+\frac{1}{x}\right)^4 + C}$

6) $\int x^2\sqrt{1-x}\,dx$

$u = 1-x$
$du = -dx$
$-du = dx$
$-u+1 = x$

$\int x^2(1-x)^{1/2}dx$

$\int (-u+1)^2(u)^{1/2}(-du)$

$-1\int (u^2-2u+1)(u^{1/2})\,du$

$-1\int u^{5/2} - 2u^{3/2} + u^{1/2}\,du$

$-1\left[\frac{2}{7}u^{7/2} - \frac{2}{5/2}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$

$-\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$\boxed{-\frac{2}{7}(1-x)^{7/2} - \frac{4}{5}(1-x)^{5/2} + \frac{2}{3}(1-x)^{3/2} + C}$

7) $\int \sqrt{9-x^2}(-2x)dx$

$u = 9-x^2$
$du = -2x\,dx$

$\int (9-x^2)^{1/2}(-2x\,dx)$

$\int u^{1/2}\,du$

$\frac{2}{3}u^{3/2}+C$

$\boxed{\frac{2}{3}(9-x^2)^{3/2}+C}$

8) $\int x^3\sqrt{x^2+1}dx$

$u = x^2+1$
$du = 2x\,dx$
$u-1 = x^2$

$\int x^3(x^2+1)^{1/2}dx$

$\frac{1}{2}\int x^2(x^2+1)^{1/2}(2x\,dx)$

$\frac{1}{2}\int (u-1)(u)^{1/2}\,(du)$

$\frac{1}{2}\int (u^{3/2}-u^{1/2})\,du$

$\frac{1}{2}\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]+C$

$\boxed{\frac{1}{5}(x^2+1)^{5/2}-\frac{1}{3}(x^2+1)^{3/2}+C}$

9) $\int \frac{x}{(1-x^2)^3}dx$

$u = 1-x^2$
$du = -2x\,dx$

$\frac{-1}{2}\int (-2x)(1-x^2)^{-3}(dx)$

$-\frac{1}{2}\int u^{-3}\,du$

$-\frac{1}{2}\left[-\frac{1}{2}u^{-2}\right]+C$

$\frac{1}{4}u^{-2}+C$

$\boxed{\frac{1}{4(1-x^2)^2}+C}$

10) $\int \frac{x}{\sqrt{4x-1}}dx$

$u = 4x-1$
$du = 4dx$
$\frac{1}{4}du = dx$
$\frac{u+1}{4} = x$

$\int x(4x-1)^{-1/2}dx$

$\int \left(\frac{u+1}{4}\right)(u)^{-1/2}\left(\frac{1}{4}du\right)$

$\frac{1}{16}\int (u+1)(u^{-1/2})\,du$

$\frac{1}{16}\int (u^{1/2}+u^{-1/2})\,du$

$\frac{1}{16}\left[\frac{2}{3}u^{3/2}+2u^{1/2}\right]+C$

$\boxed{\frac{1}{24}(4x-1)^{3/2}+\frac{1}{8}(4x-1)^{1/2}+C}$

#'s 11 – 12: Evaluate the definite integral.

11) $\int_0^4 \frac{1}{\sqrt{2x+1}}dx$

$u = 2x+1$
$du = 2dx$

$2(4)+1$
$2(0)+1$

$\frac{1}{2}\int_0^4 (2x+1)^{-1/2}(dx)$

$\frac{1}{2}\int_1^9 u^{-1/2}\,du$

$\frac{1}{2}(2u^{1/2})\Big]_1^9$

$u^{1/2}\Big]_1^9$

$(9)^{1/2}-(1)^{1/2}$

$3-1 = \boxed{2}$

12) $\int_0^{\pi/2}\cos\left(\frac{2x}{3}\right)dx$

$u = \frac{2}{3}x$
$du = \frac{2}{3}dx$

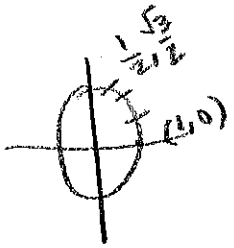
$\frac{2}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{3}$
$\frac{2}{3}(0) = 0$

$\frac{3}{2}\int_0^{\pi/3}\cos(u)\,du$

$\frac{3}{2}(\sin(u))\Big]_0^{\pi/3}$

$\frac{3}{2}\sin\left(\frac{\pi}{3}\right)-\frac{3}{2}\sin(0)$

$\frac{3}{2}\left(\frac{\sqrt{3}}{2}\right)-\frac{3}{2}(0)$



$\boxed{\frac{3\sqrt{3}}{4}}$