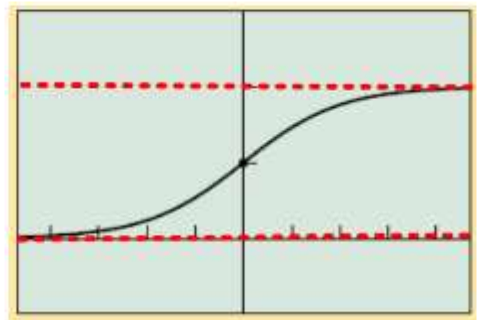
**AP Calculus BC**
**Unit 8 – Integration Techniques**

# Day 10 Notes: Logistics Differential Equations



## General Logistic Formula:

$$P = \frac{M}{1 + Ae^{-(Mk)t}}$$

M = carrying capacity
k = growth constant

**Example 1:**

Let $f(x) = \frac{50}{1+4e^{-0.2x}}$

a) Find where f(x) is continuous.

b) Find $\lim_{x \to \infty} f(x)$.

c) Find $\lim_{x \to -\infty} f(x)$.

d) Find the y-intercept of the graph of f(x).

e) Find all horizontal asymptotes of the graph of f(x).

f) Find the carrying capacity of f(x).

## Logistics Differential Equation:

$$\frac{dP}{dt} = kP(M - P)$$

***Things to remember about logistics differential equations:***

1) *M = carrying capacity*
2) *No matter what the initial population is,* $\lim_{t \to \infty} P(t) = M$.
3) *The growth rate of P(t) is increasing fastest when* $P = M/2$.
4) *You can solve a logistics differential equation by separating the variables and using partial fractions.*

**Example 2:** The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation $dP/dt = 0.008P(100 - P)$, where t is measured in years.

a) What is the carrying capacity for bears in this wildlife preserve?

b) What is the bear population when the population is growing the fastest?

c) What is the rate of change of the population when it is growing the fastest?

d) If P(0) = 20, what is $\lim_{t \to \infty} P(t)$?

**Example 3:** In 1985 and 1987, the Michigan Department of Natural Resources airlifted 61 moose from Algonquin Park, Ontario to Marquette County in the Upper Peninsula. It was originally hoped that the population P would reach carrying capacity in about 25 years with a growth rate of

$$\frac{dP}{dt} = 0.0003P(1000 - P)$$

a) According to the model, what is the carrying capacity?

b) Solve the differential equation with the initial condition P(0) = 61.

**Example 4:** A 2000-gallon tank contains guppies. The rate of growth of the guppies in the tank is $\frac{dP}{dt} = P(0.225 - 0.0015P)$, where t is in weeks.

a) Without actually solving the differential equation, what is $\lim_{t \to \infty} P(t)$? Interpret the limit in the context of the problem.

b) Solve the differential equation, given that P(0) = 20.

**AP Calculus BC** **Name: ______________________________**
**Unit 8 – Day 10 – Assignment**

1) If $\frac{dy}{dt} = 2y(12 - 3y)$, then the maximum value of y is

a) y = 1 b) y = 2 c) y = 4 d) y = 12 e) y has no max value

2) What are all of the horizontal asymptotes of all the solutions of the logistics differential equation $\frac{dy}{dx} = y\,(8 - 0.001y)$?

a) y = 0 b) y = 8 c) y = 8000 d) y = 0 & y = 8 e) y = 0 & y = 8000

3) What is the carrying capacity for a population whose growth rate is modeled by $\frac{dP}{dt} = 6P - 0.012P^2$?

a) 500 b) 50 c) 0.012 d) 0.002 e) None of these

4) A rumor spreads through a community at a rate of $\frac{dy}{dt} = 2y(1 - y)$, where y is the proportion of the populations that has heard the rumor at time t.

a) What proportion of the community has heard the rumor when it is spreading the fastest?

b) If, at time t = 0, 10 percent of the community has heard the rumor, find y as a function of t.

5) A population is modeled by a function P that satisfies the logistics differential equation $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right)$.

a) If P(0) = 3, what is $\lim_{t \to \infty} P(t)$?
If P(0) = 20, what is $\lim_{t \to \infty} P(t)$?

b) If P(0) = 3, for what value of P is the population growing the fastest?

c) A different population is modeled by the function Y that satisfies the separable differential equation $\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right)$.
Find Y(t) if Y(0) = 3.