AP Calculus BC
Unit 8 – Day 10 – Assignment

Name: Answer Key*

↓ pull out

1) If $\frac{dy}{dt} = 2y(\underset{3}{\overset{(3)}{12}} - \underset{3}{3}y)$, then the maximum value of y is $6y(4-y)$ $M=4$

a) y = 1 b) y = 2 c) y = 4 d) y = 12 e) y has no max value

2) What are all of the horizontal asymptotes of all the solutions of the logistics differential equation $\frac{dy}{dx} = y(\underset{.001}{8} - \underset{.001}{0.001}y)$? $.001y(8000-y)$ $M=8000$

a) y = 0 b) y = 8 c) y = 8000 d) y = 0 & y = 8 e) y = 0 & y = 8000

3) What is the carrying capacity for a population whose growth rate is modeled by $\frac{dP}{dt} = \underset{.012}{6P} - \underset{.012}{0.012}P^2$? $.012P(500-P)$

a) 500 b) 50 c) 0.012 d) 0.002 e) None of these

4) A rumor spreads through a community at a rate of $\frac{dy}{dt} = 2y(1-y)$, where y is the proportion of the populations that has heard the rumor at time t.

a) What proportion of the community has heard the rumor when it is spreading the fastest?

$M = 1 \quad \frac{1}{2} = \boxed{\frac{1}{2}}$

b) If, at time t = 0, 10 percent of the community has heard the rumor, find y as a function of t.

$$\frac{dy}{dt} = 2y(1-y)$$

$$\int \frac{dy}{y(1-y)} = \int 2\,dt$$

$$\int \frac{1}{y} + \frac{1}{1-y}\,dy = \int 2\,dt$$

$$\frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}$$

$$1 = A(1-y) + B(y)$$

$\underline{y=1}$: $1 = A(\cancel{1-1}) + B(1)$, $B=1$

$\underline{y=0}$: $1 = A(1-0) + \cancel{B(0)}$, $A=1$

$$\ln|y| - \ln|1-y| = 2t + c$$

$$\ln(1-y) - \ln(y) = -2t - c$$

$$\ln\left(\frac{1-y}{y}\right) = -2t - c$$

$$\frac{1-y}{y} = e^{-2t}e^{-c}$$

$$\frac{1}{y} - 1 = e^{-2t}e^{-c}$$

$$\frac{1}{y} = 1 + e^{-2t}e^{-c}$$

$y = 10$ & $t = 0$

$$\frac{1}{10} = 1 + \cancel{e^{-2(0)}}e^{-c}$$

$$-.9 = e^{-c}$$

$$\frac{1}{y} = 1 + -.9e^{-2t}$$

$$\boxed{y = \frac{1}{1 + -.9e^{-2t}}}$$

5) A population is modeled by a function P that satisfies the logistics differential equation

$\frac{dP}{dt} = \frac{1}{12}\frac{P}{5}(12)(1 - \frac{P}{12})(12)$ $\qquad \frac{P}{60}(12 - P)$

a) If P(0) = 3, what is $\lim_{t \to \infty} P(t)$? $\boxed{12}$

If P(0) = 20, what is $\lim_{t \to \infty} P(t)$? $\boxed{12}$

b) If P(0) = 3, for what value of P is the population growing the fastest?

$$\frac{12}{2} = \boxed{6}$$

c) A different population is modeled by the function Y that satisfies the separable differential equation $\frac{dY}{dt} = \frac{Y}{5}(1 - \frac{t}{12})$.
Find Y(t) if Y(0) = 3.

Different than logistic b/c Y & t on same side

$$\frac{dy}{dt} = \frac{1}{60} Y (12 - t)$$

$$\int \frac{1}{Y} dY = \int \frac{1}{60}(12 - t)\, dt$$

$$\ln|y| = \int \frac{1}{5} - \frac{1}{60} t \, dt$$

$$\ln|y| = \frac{1}{5}t - \frac{\frac{1}{60}t^2}{2} + c$$

$$\ln|y| = \frac{1}{5}t - \frac{1}{120}t^2 + c$$

$$y = e^{\frac{1}{5}t - \frac{1}{120}t^2} \cdot e^c$$

$$Y(0) = 3$$

$$3 = e^{\frac{1}{5}(0) - \frac{1}{12}(0)^2}$$

$$3 = e^0 e^c$$

$$3 = e^c$$

$$\boxed{y = 3e^{\frac{1}{5}t - \frac{1}{120}t^2}}$$

~~d) For the function Y found in part (c), what is $\lim Y(t)$~~?