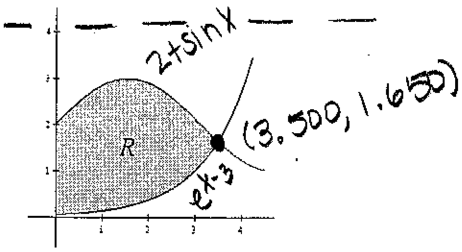
AP Calculus AB - Unit 7 - Review - BINGO

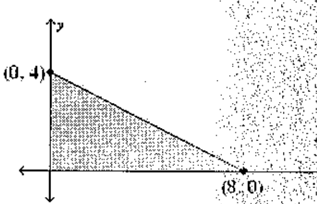


Let R be the region in the first quadrant bounded by the graphs of $y = 2 + \sin x$, $y = e^{x-3}$, and the y – axis as shown in the figure above. Find the volume of the solid generated when R is rotated around the line $\boxed{y = 4.}$ (Calculator)

$$V = \pi \int_0^{3.500} [e^{x-3} - 4]^2 - [2 + \sin x - 4]^2 \, dx$$

$$\boxed{V = 115.380}$$

The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis and the graph of the line $x + 2y = 8$, as shown in the figure below. If the cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid? (Calculator)



$x + 2y = 8$

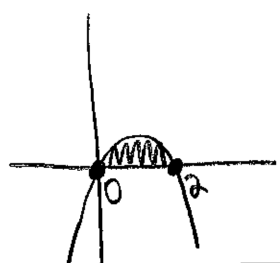
$2y = 8 - x$

$y = 4 - \frac{1}{2}x$

$$V = \frac{\pi}{8} \int_0^8 [4 - \tfrac{1}{2}x - 0]^2 \, dx$$

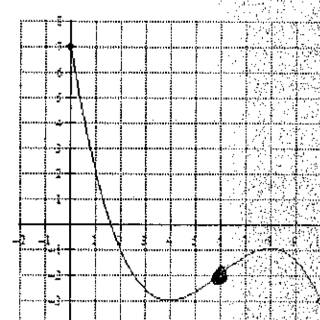
$$\boxed{= 16.755}$$

The region bounded by the graph of $y = 2x - x^2$ and the x – axis is the base of a solid. For this solid, each cross section perpendicular to the x – axis is an equilateral triangle. What is the volume of this solid? (Calculator)



$$V = \frac{\sqrt{3}}{4} \int_0^2 [2x - x^2 - 0]^2 \, dx = \boxed{0.462}$$

The graph of a function f is shown in the figure below and has a horizontal tangent at $x = 4$ and $x = 8$. If $g(x) = x^2 - \int_0^{2x} f(t)\, dt$, what is the value of $g'(3)$? (No Calculator)



$g'(x) = 2x - f(2x)(2)$

$g'(3) = 2(3) - f(2 \cdot 3)(2)$

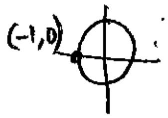
$= 6 - f(6)(2)$

$= 6 - (-2)(2)$

$= 6 + 4 \quad = \boxed{10}$

What is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$. (No Calculator)

$\int dy = \int 2\sin x \, dx$



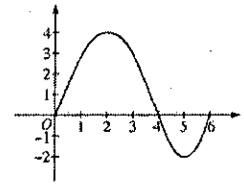
$y = -2\cos x + C$

$1 = -2\cos(\pi) + C$

$1 = -2(-1) + C$

$-1 = C$

$\boxed{y = -2\cos x - 1}$



Graph of f

The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection? (No Calculator)

$g'(x) = f(x)(1)$

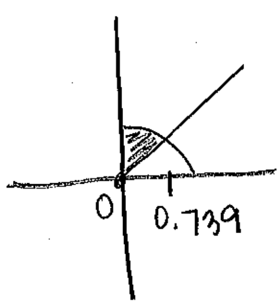
$g''(x) = f'(x)$

↑

changes signs → rel. max/min

$\boxed{x = 2 \ \& \ x = 5}$

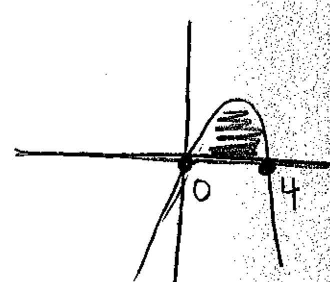
Find the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$ and the y – axis. (Calculator)



$A = \int_0^{0.739} [\cos x - x]\,dx$

$= \boxed{0.400}$

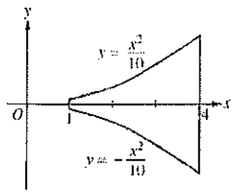
Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 4x - x^2$ and $y = 0$ about the $\boxed{x\text{ – axis.}}$ (Calculator)

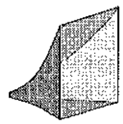


$V = \pi \int_0^4 [4x - x^2 - 0]^2\,dx$

$= \boxed{107.233}$

The base of a loud speaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$ as shown in the figures to the right. For this loud speaker, the cross sections perpendicular to the x – axis are squares. What is the volume of this speaker, in cubic units? (Calculator)



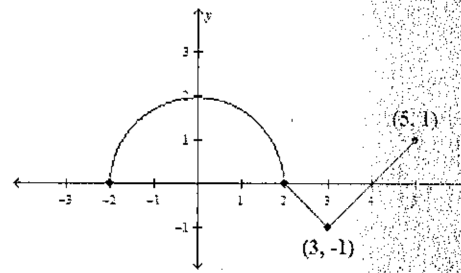


$$V = \int_1^4 \left[\frac{x^2}{10} - \left(-\frac{x^2}{10} \right) \right]^2 dx$$

$$= \boxed{8.184}$$

$$G(x) = x^2 + \int_{-2}^{2x} f(t)\,dt$$

What is the value of $G(2)$? (No Calculator)



Graph of f

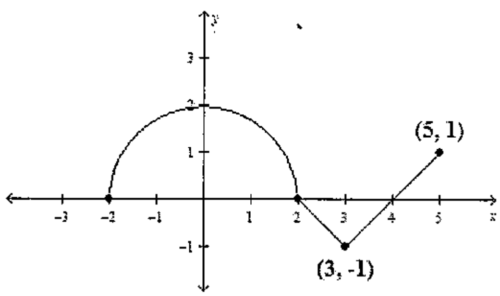
$$G(2) = (2)^2 + \int_{-2}^{2(2)} f(t)\,dt$$

$$= 4 + \int_{-2}^{4} f(t)\,dt$$

$$= 4 + \tfrac{1}{2}\pi(2)^2 - \tfrac{1}{2}(2)(1)$$

$$= \boxed{3 + 2\pi}$$

Find the value of. $G'(2)$.
(No Calculator.)



Graph of f

$$G(x) = x^2 + \int_{-2}^{2x} f(t)\,dt$$

$$G'(x) = 2x + f(2x)(2)$$

$$G'(2) = 2(2) + f(2\cdot 2)(2)$$

$$= 4 + f(4)(2)$$

$$= 4 + (0)(2) = \boxed{4}$$

If $g(x) = \int_1^{x^2} \frac{3t}{t^3+1}\,dt$, then what is the value of $g'(2)$? (No Calculator)

$$g'(x) = \frac{3(x^2)}{(x^2)^3 + 1}(2x)$$

$$g'(x) = \frac{3x^2}{x^6+1}(2x)$$

$$g'(2) = \frac{3(2)^2}{(2)^6+1}(2\cdot 2)$$

$$= \frac{12}{65}(4)$$

$$= \boxed{48/65}$$

If $g(x)=\int_0^x t^3e^t\,dt$, find $g''(1)$.

(No Calculator)

$g'(x) = x^3e^x$

$g''(x) = (3x^2)(e^x) + (x^3)(e^x)$

$g''(1) = (3(1)^2)(e^1) + (1)^3(e^1)$

$= 3e + e$

$= \boxed{4e}$

$$\int \frac{2x^2}{\sqrt{x^3-2}}\,dx =$$

(No Calculator)

$3x^2$

$\frac{1}{3}\cdot 2\int 3(\cancel{2}x^2)(x^3-2)^{-1/2}\,dx$

$\frac{2}{3}\left[\frac{(x^3-2)^{1/2}}{1/2}\right] + C$

$\frac{2}{3}\cdot\frac{2}{1}$

$\frac{4}{3}$

$\boxed{\frac{4}{3}\sqrt{x^3-2} + C}$

If $\frac{dy}{dx}=\frac{x^2}{y}$ and $f(0) = -4$, find the particular solution to the differential equation.

(No Calculator)

$\int y\,dy = \int x^2\,dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$

$\frac{1}{2}(-4)^2 = \cancel{\frac{1}{3}(0)^3} + C$

$8 = C$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 + 8$

$y^2 = \frac{2}{3}x^3 + 16$

$y = \pm\sqrt{\frac{2}{3}x^3+16}$

$\boxed{y = -\sqrt{\frac{2}{3}x^3+16}}$

-4

$$-\frac{1}{4}\int_0^1 4e^{-4x}\,dx$$

(No Calculator)

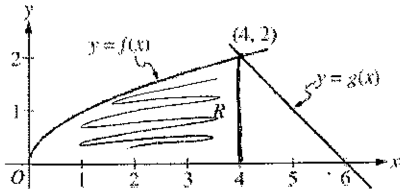
$-\frac{1}{4}\left[e^{-4x}\right]_0^1$

$-\frac{1}{4}\left[e^{-4(1)} - e^{-4(0)}\right]$

$-\frac{1}{4}\left[e^{-4} - e^{0}\right]$

$-\frac{1}{4}\left[e^{-4} - 1\right] = -\frac{1}{4}e^{-4} + \frac{1}{4}$

$= \boxed{.25 - .25e^{-4}}$



Region R is the region in the first quadrant bounded by the graphs of $f(x) = \sqrt{x}$, $g(x) = 6 - x$ and the x – axis. Find the area of R. (No Calculator)

$$A = \int_0^4 x^{1/2}\,dx + \int_4^6 6 - x\,dx$$

$\sqrt{4^3}$ $\quad \left.\frac{x^{3/2}}{3/2}\right]_0^4 + \left. 6x - \frac{1}{2}x^2 \right]_4^6$

$$\frac{2}{3}(4)^{3/2} - \cancel{\frac{2}{3}(0)^{3/2}} + \left[6(6) - \frac{1}{2}(6)^2 - \left[6(4) - \frac{1}{2}(4)^2\right]\right]$$

$\frac{2}{3}(8)$

$$\frac{16}{3} + 36 - 18 - 24 + 8 = \boxed{\frac{22}{3}}$$

$$\int \frac{x}{x^2 - 4}\,dx =$$

(No Calculator)

$2x$

$$\frac{1}{2}\int 2x\left((x^2-4)^{-1}\right)dx$$

$$\boxed{\frac{1}{2}\ln|x^2 - 4| + C}$$

If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\,dt$, then $f'(2) =$

(No Calculator)

$$f'(x) = \sqrt{(2x)^2 - (2x)}\;(2)$$

$$f'(2) = \sqrt{(2 \cdot 2)^2 - (2 \cdot 2)}\;(2)$$

$$= \sqrt{16 - 4}\;(2)$$

$$= \sqrt{12}\;(2)$$

$$= \boxed{2\sqrt{12}}$$