AP Calculus AB
Unit 7 – Review

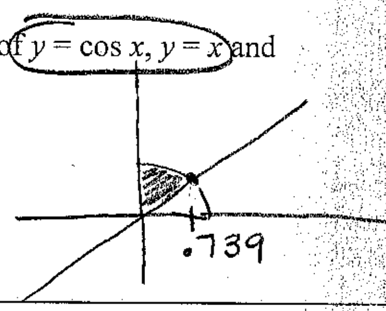
Name: Answer Key*

calc.

1. Find the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$ and the y – axis.

math9

$$A = \int_0^{.739} \cos x - x \, dx = \boxed{0.400}$$

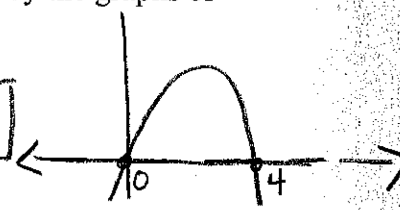


2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 4x - x^2$ and $y = 0$ about the x – axis.

calc.

math9

$$V = \pi \int_0^4 [4x - x^2 - 0]^2 - [0-0]^2 dx = \boxed{107.233}$$



3. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, the x – axis, and the y – axis is revolved about the x – axis. Its volume is found by evaluating which of the following integrals?

No calc

A. $\pi \int_1^8 (x^2 + 1)^3 \, dx$

B. $\pi \int_1^8 (x^2 + 1)^6 \, dx$

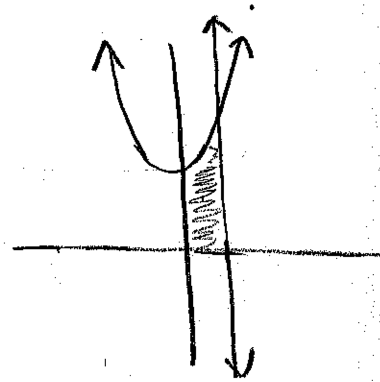
C. $\pi \int_0^1 (x^2 + 1)^3 \, dx$

D. $\pi \int_0^1 (x^2 + 1)^6 \, dx$

E. $2\pi \int_0^1 (x^2 + 1)^6 \, dx$

$$V = \pi \int_0^1 [(x^2+1)^3 - 0]^2 \, dx$$

$$= \pi \int_0^1 (x^2+1)^6 \, dx$$



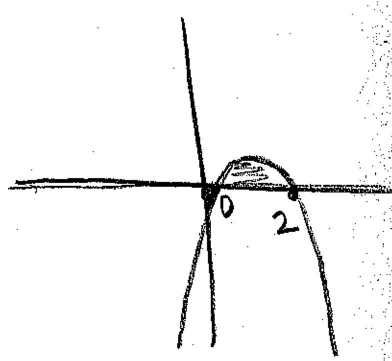
y=0

4. The region bounded by the graph of $y = 2x - x^2$ and the x – axis is the base of a solid. For this solid, each cross section perpendicular to the x – axis is an equilateral triangle. What is the volume of this solid?

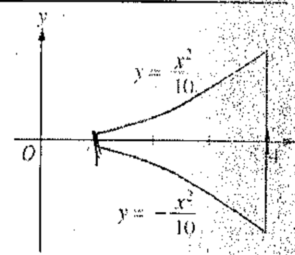
calc

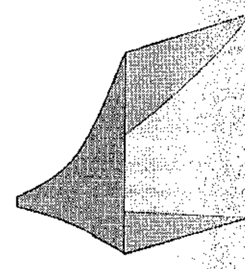
$$V = \frac{\sqrt{3}}{4} \int_0^2 [2x - x^2 - 0]^2 dx = \boxed{0.462}$$

math9



5. The base of a loud speaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$ as shown in the figures to the right. For this loud speaker, the cross sections perpendicular to the x – axis are squares. What is the volume of this speaker, in cubic units?





calc

math 9

$$V = \int_1^4 \left[\frac{x^2}{10} - \left(-\frac{x^2}{10}\right)\right]^2 dx = \boxed{8.184}$$

6. The slope field pictured below represents all general solutions to which of the following differential equations?

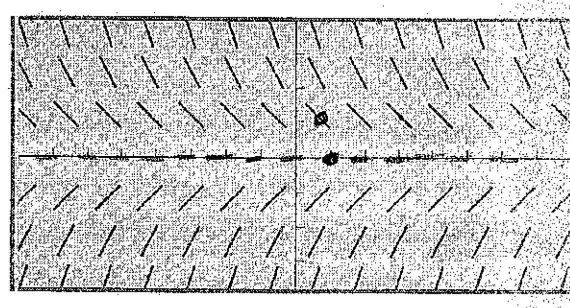
~~A.~~ $\frac{dy}{dx} = 2x$ $\quad 2(1) = 2 \quad (1,0)$ is 0.

~~B.~~ $\frac{dy}{dx} = -2x$ $\quad -2(1) = -2$

C. $\frac{dy}{dx} = -y$ $\quad 0 \quad -1$ ✓ $(1,1)$

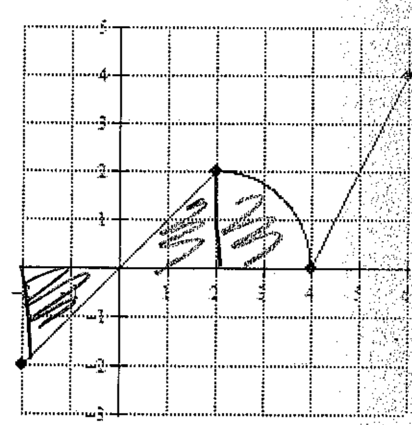
D. $\frac{dy}{dx} = y$ $\quad 0 \quad 1$

~~E.~~ $\frac{dy}{dx} = x + y$ $\quad 1 + 0 = 1$



no calc

7. The graph of a function f, which consists of two line segments and a quarter circle, is pictured to the right. If $H(x) = \int_{-2}^{x} f(t)\,dt$, which of the following statements is true?



$H'(x) = f(x)$

$H''(x) = f'(x)$

A. $H(4) < H'(2) < H''(3)$

B. $H(4) < H''(3) < H'(2)$

C. $H'(2) < H(4) < H''(3)$

D. $H''(3) < H(4) < H'(2)$

E. $H''(3) < H'(2) < H(4)$

no calc

$$H(4) = \int_{-2}^{4} f(t)\,dt = -\tfrac{1}{2}\cancel{(2)}(2) + \tfrac{1}{2}(2)\cancel{(2)} + \tfrac{1}{4}\pi(2)^2 = \pi$$

$H'(2) = f(2) = 2$

$H''(3) = f'(3) = -\#$

$H''(3) < H'(2) < H(4)$

$u^2 = x$

$2u\,du = dx$

8. Using the substitution $u = \sqrt{x}$, $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

A. $2\int_1^{81} e^u\,du$

B. $2\int_1^{9} e^u\,du$

C. $2\int_1^{3} e^u\,du$

D. $\frac{1}{2}\int_1^{3} e^u\,du$

E. $\int_1^{9} e^u\,du$

$= \int_1^3 \frac{e^u}{\cancel{u}}(2\cancel{u}\,du)$

$= 2\int_1^3 e^u\,du$

no calc

deriv. of $-3x$ is -3

9. $-\frac{1}{3}\int_0^1 -3e^{-3x}\,dx =$

$-\frac{1}{3}\left[e^{-3x}\right]_0^1 = -\frac{1}{3}\left[e^{-3(1)} - e^{-3(0)}\right]$

$= -\frac{1}{3}\left[e^{-3} - 1\right]$

$= \boxed{-\frac{1}{3}e^{-3} + \frac{1}{3}}$

no calc

2nd Fund. Thm of Calc.

10. If $g(x) = \int_1^{2x} \frac{3t}{t^3+1}\,dt$, then what is the value of $g'(2)$?

$g'(x) = \frac{3(2x)}{(2x)^3+1}(2)$

$g'(x) = \frac{12x}{8x^3+1}$

$g'(2) = \frac{12(2)}{8(2)^3+1}$

$= \boxed{\frac{24}{65}}$

no calc

11. $\int \frac{2x^2}{x^3-2}\,dx = \frac{1}{3}\cdot 2\int 3x^2(x^3-2)^{-1}\,dx$

$3x^2$

$\frac{2}{3}\left[\frac{(x^3-2)^0}{0}\right] + C$

$\boxed{\frac{2}{3}\ln|x^3-2| + C}$

no calc

Free Response #1 – Calculator NOT Permitted

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b. Write an equation of the tangent line to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$. ~~Explain why the tangent line gives a good approximation of $f(1.1)$.~~

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

(b) point $(1,-1)$

slope $\rightarrow \frac{dy}{dx} = -\frac{2(1)}{-1} = \frac{-2}{-1} = 2$

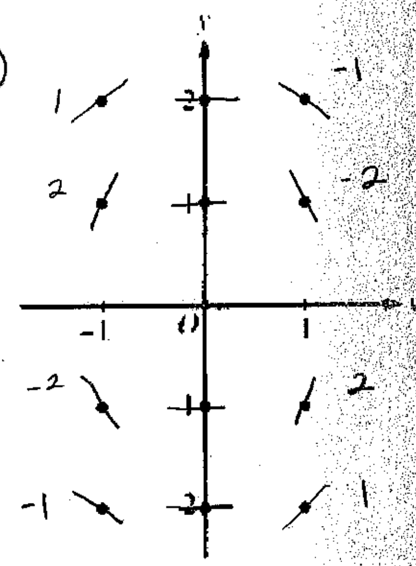
$$\boxed{y+1 = 2(x-1)}$$

$f(1.1) \approx \quad y+1 = 2(1.1-1)$

$y+1 = 0.2$

$$\boxed{y = -0.8}$$

(a)



(c) $\frac{dy}{dx} = -\frac{2x}{y}$

$\int y\,dy = \int -2x\,dx$

$\frac{1}{2}y^2 = -x^2 + C$

$f(1) = -1$

$\frac{1}{2}(-1)^2 = -(1)^2 + C$

$+\frac{1}{2} = -1 + C$

$C = 3/2$

$2\left(\frac{1}{2}y^2 = -x^2 + \frac{3}{2}\right)$

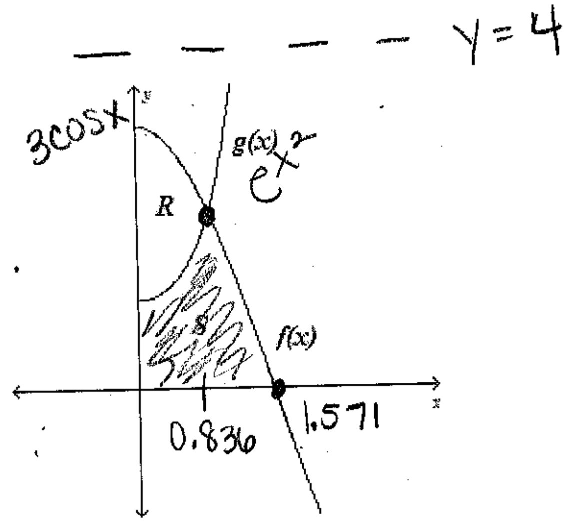
$\sqrt{y^2} = \pm\sqrt{-2x^2 + 3}$

$y = \pm\sqrt{-2x^2 + 3}$

$f(1) = -1$

$$\boxed{f(x) = -\sqrt{-2x^2+3}}$$

Free Response #2 – Calculator Permitted



Let R be the region in the first quadrant bounded by the y – axis and the graphs of $f(x) = 3\cos x$ and $g(x) = e^{x^2}$. Let S be the region in the first quadrant bounded by the graphs of $f(x)$, $g(x)$ and the x – axis.

a. Find the area of region S.

$$A = \int_0^{1.571} 3\cos x\,dx - \int_0^{.836} (3\cos x - e^{x^2})\,dx = 3.000 - 1.146 \doteq \boxed{1.854}$$

whole − R

b. Region R is rotation about the line $y = 4$. Find the volume of the solid generated.

$$V = \pi \int_0^{.836} [e^{x^2} - 4]^2 - [3\cos x - 4]^2\,dx = \boxed{14.555}$$

c. Region R is the base of a solid whose cross sections are equilateral triangles. Find the volume of this solid.

$$V = \frac{\sqrt{3}}{4} \int_0^{0.836} [3\cos x - e^{x^2}]^2\,dx = \boxed{0.804}$$