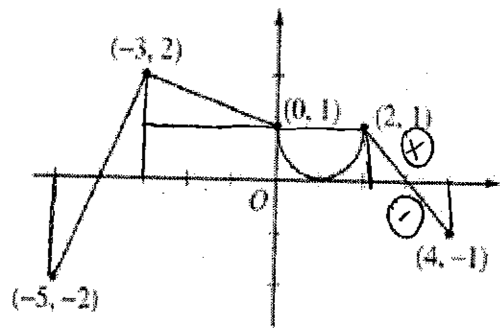
**AP Calculus AB**
**Unit 7 – Quiz Review (Days 1 – 3)**

**Name:** Answer Key*



Graph of $f$

The graph of a function, $f$, which consists of three line segments and a semi-circle is pictured above. Let $g(x)=\int_{-3}^{x} f(t)dt$. Use this information to answer questions 1 – 4.

1. Compute the values of $g(-5)$ and $g(4)$.

$g(-5) = \int_{-3}^{-5} f(t)dt = -\int_{-5}^{-3} f(t)dt \overset{\text{area}}{=} -\left[-\frac{1}{2}(1)(2)+\frac{1}{2}(1)(2)\right]$

$= -[0] = \boxed{0}$

$g(4) = \int_{-3}^{4} f(t)dt = (1)(3)+\frac{1}{2}(1)(3)+\left[2(1)-\frac{1}{2}\pi(1)^2\right]+\frac{1}{2}(1)(1)-\frac{1}{2}(1)(1) = 3+1.5+\left[2-\frac{1}{2}\pi\right]+\cancel{.5}-\cancel{.5}$

$= \boxed{6.5-\frac{1}{2}\pi}$

2. Find $g'(2)$ and $g''(2)$. Show or explain your work.

$g'(x) = f(x)(1) \rightarrow g''(x) = f'(x)$

$g'(2) = f(2) = \boxed{1}$

$g''(2) = f'(2)$

undefined b/c $f(x)$ has a cusp at $x=2$

3. Find the coordinates of the absolute maximum of $g$ on the closed interval $[-5, 4]$. Justify your answer.

[Extreme Value Thm]

$g(-5) = 0$ (see #1)

$g(4) = 6.5-\frac{1}{2}\pi$ (see #1)

$g'(x) = f(x)$

$g'(x)$ has a rel. max when $f(x)$ changes from $\oplus \rightarrow \ominus$ at $\rightarrow x=3$

$g(3) = \int_{-3}^{3} f(t)dt \overset{\text{area}}{=} (3)(1)+\frac{1}{2}(1)(3)+\left[2-\frac{1}{2}\pi(1)^2\right]+\frac{1}{2}(1)(1) = 3+1.5+2-\frac{1}{2}\pi+.5$

$7-\frac{1}{2}\pi$

$\boxed{\text{abs. max at } (3, 7-\frac{1}{2}\pi)}$

4. ~~The second derivative of g is not defined at x = −3, x = 0 and x = 2.~~ W~~hich of these three values is/are coordinates of~~ points of inflection of the graph of $g$? Justify your answer.

• $g(x)$ has point of infl. when $g''(x)$ changes signs

$g'(x) = f(x) \rightarrow g''(x) = f'(x)$

• $g(x)$ has P.O.I when $f'(x)$ changes signs.

↳ $f(x)$ changes from incr → decr or decr → incr. (rel max/min)

$\boxed{x=-3}$ $\boxed{x=1}$ $\boxed{x=2}$

5. If $\frac{dy}{dx} = \frac{x^3}{y}$ and f(0) = 2, find the particular solution to the differential equation.

$\int y\,dy = \int x^3\,dx$

$\frac{1}{2}y^2 = \frac{1}{4}x^4 + C$

$f(0) = 2$

$\frac{1}{2}(2)^2 = \frac{1}{4}\cancel{(0)^4} + C$

$2 = C$

$\frac{1}{2}y^2 = \frac{1}{4}x^4 + 2$

$y^2 = \frac{1}{2}x^4 + 4$

$y = \pm\sqrt{\frac{1}{2}x^4 + 4}$

$f(0) = 2$

$\sqrt{\frac{1}{2}(0)^4 + 4} = 2$ ✓

$\boxed{f(x) = \sqrt{\frac{1}{2}x^4 + 4}}$

6. If $u = 2x - 1$, then $\int x\sqrt[3]{2x - 1}\,dx =$

$u = 2x - 1$

$1\,du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{u+1}{2} = x$

$\int x(2x-1)^{1/3}\,dx$

$\int \left(\frac{u+1}{2}\right)(u)^{1/3}\left(\frac{du}{2}\right)$

$\int \frac{1}{4}(u+1)(u^{1/3})\,du$

$\frac{1}{4}\int u^{4/3} + u^{1/3}\,du$

$\frac{1}{4}\left[\frac{u^{7/3}}{7/3} + \frac{u^{4/3}}{4/3}\right] + C$

$\frac{1}{4}\cdot\frac{3}{7} = \frac{3}{28}$ $\quad \frac{1}{4}\cdot\frac{3}{4} = \frac{3}{16}$

$\frac{3}{28}u^{7/3} + \frac{3}{16}u^{4/3} + C$

$\boxed{\frac{3}{28}(2x-1)^{7/3} + \frac{3}{16}(2x-1)^{4/3} + C}$

7. $\int 7x\sqrt{4x^2 - 3}\,dx$ $\quad 8x$

$\frac{1}{8}\cdot 7 \int 8x\left((4x^2-3)^{1/2}\right)dx$

$\frac{7}{8}\left[\frac{(4x^2-3)^{3/2}}{3/2}\right] + C$

$\frac{7}{8}\cdot\frac{2}{3}$

$\frac{14}{24} = \frac{7}{12}$

$\boxed{\frac{7}{12}(4x^2-3)^{3/2} + C}$