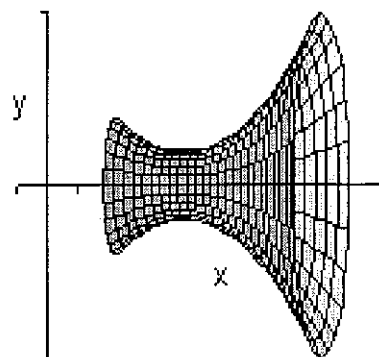
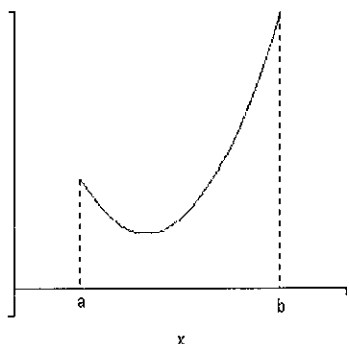


AP Calculus
Unit 7 – Advanced Integration & Applications

Day 6 Notes: Volume of Solids of Revolution

A solid of revolution is formed when a flat, two-dimensional shape is rotated around an axis. Consider the flat region below to the left. When that region is rotated about the x -axis, the solid pictured below to the right is formed. This objective of this lesson is to learn to find the volume of such a solid.



Now, imagine slicing the solid into individual discs of height 1 unit. The volume of one of those discs is
cylinder $\rightarrow V = \pi r^2 h$, or $V = \pi r^2 (1)$

$$\text{Sum of all the discs} = \int_a^b \pi [f(x) - \text{Axis of Rotation}]^2 dx$$

$$\text{Volume of the Solid} = \pi \int_a^b [f(x) - \text{Axis of Rotation}]^2 dx$$

DISK method

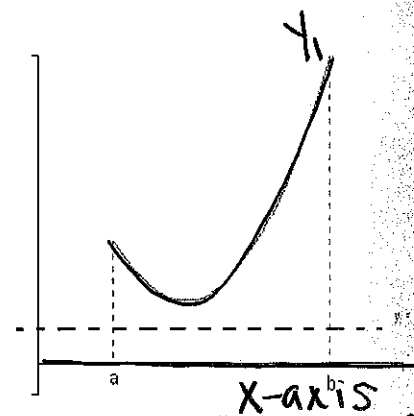
Notice that the axis of rotation is the x -axis and the bottom function of the region is also the x -axis. Imagine for a moment what the solid would look like if the axis of rotation were still the x -axis but the bottom function of the region was the line $y=c$. The solid would look similar except for the fact that there would be a cylinder that is cut out of the center.

To find the volume of this solid, we would find the volume of the whole solid that we found previously and then subtract out the solid in the form of a cylinder.

In order to do this, we use the formula below to find the volume of such a solid.

$$\text{Volume} = \pi \int_a^b [\text{OuterFunction} - \text{axis}]^2 - [\text{InnerFunction} - \text{axis}]^2 dx$$

The “outer function” is defined to be the function that is farther from the axis of rotation. The “inner function” is defined to be the function that is closer to the axis of rotation.



Rotate around $y=c$

$$V = \pi \int_a^b [y_1 - c]^2 - [0 - c]^2 dx$$

Washer Method

If the axis of rotation is the x -axis or is parallel to the x -axis, the integrand needs to be in terms of x and the limits of integration need to be the x -values of the points of intersection of the curves that form the region being rotated.

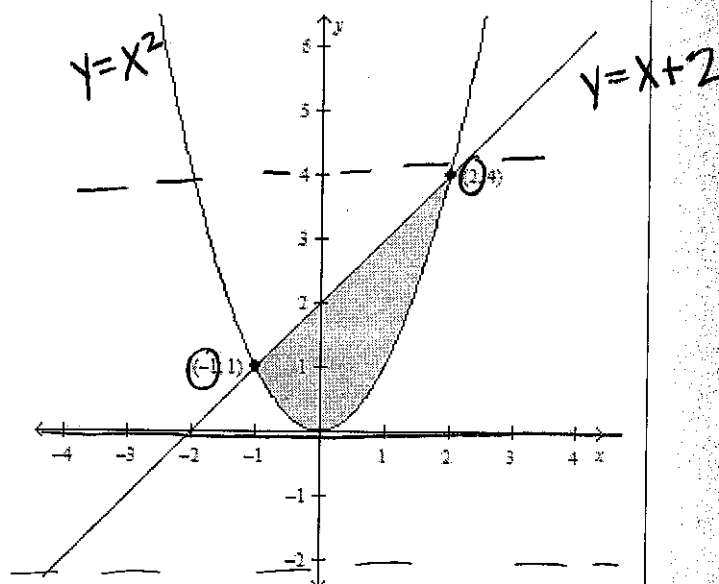
Consider the region pictured to the right that is bounded by the graphs of $y = x^2$ and $y = x + 2$.

Find the volume of the solid formed when the region is rotated about the x -axis.

$$V = \pi \int_{-1}^2 [x+2-0]^2 - [x^2-0]^2 dx$$

$$V = \pi \int_{-1}^2 [x+2]^2 - [x^2]^2 dx$$

$$V = 45.239$$



Find the volume when the region is rotated about the line $y = 4$.

$$V = \pi \int_{-1}^2 [x^2-4]^2 - [x+2-4]^2 dx$$

$$= \pi \int_{-1}^2 [x^2-4]^2 - [x-2]^2 dx$$

$$V = 67.858$$

Find the volume when the region is rotated about the line $y = -2$.

$$V = \pi \int_{-1}^2 [x+2-(-2)]^2 - [x^2-(-2)]^2 dx$$

$$= \pi \int_{-1}^2 [x+4]^2 - [x^2+2]^2 dx$$

$$V = 101.788$$

If the axis of rotation is the y -axis or is parallel to the y -axis, the integrand needs to be in terms of y and the limits of integration need to be the y -values of the points of intersection of the curves that form the region being rotated.

Consider the region pictured to the right that is bounded by the graphs $y = \pm\sqrt{x}$ and $y = x - 2$.

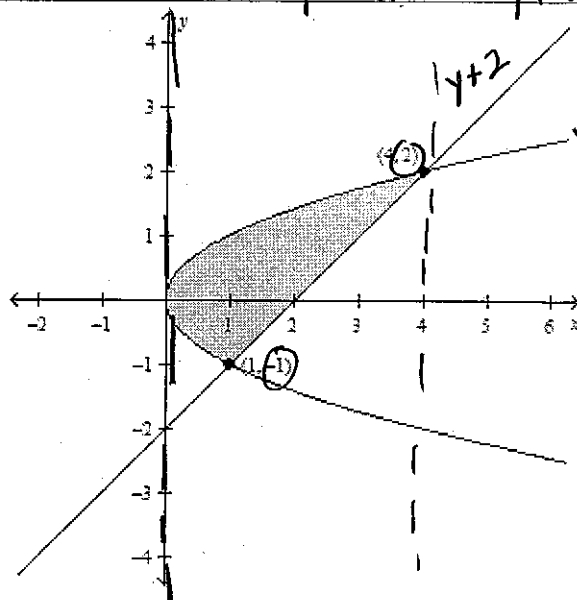
$$y^2 = x \quad y + 2 = x$$

Find the volume when the region is rotated about the y -axis.

$$V = \pi \int_{-1}^2 [(y+2)-0]^2 - [y^2-0]^2 dy$$

$$= \pi \int_{-1}^2 (y+2)^2 - (y^2)^2 dy$$

$$V = 45.239$$



Find the volume when the region is rotated about the line $x = 4$.

$$V = \pi \int_{-1}^2 [y^2 - 4]^2 - [y + 2 - 4]^2 dy$$

$$= \pi \int_{-1}^2 (y^2 - 4)^2 - (y - 2)^2 dy$$

$$V = 67.858$$

Find the volume when the region is rotated about the line $x = 7$.

$$V = \pi \int_{-1}^2 (y^2 - 7)^2 - (y + 2 - 7)^2 dy$$

$$= \pi \int_{-1}^2 (y^2 - 7)^2 - (y - 5)^2 dy$$

$$V = 152.681$$