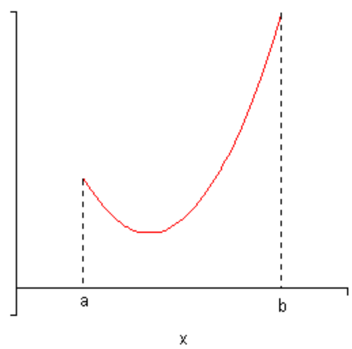
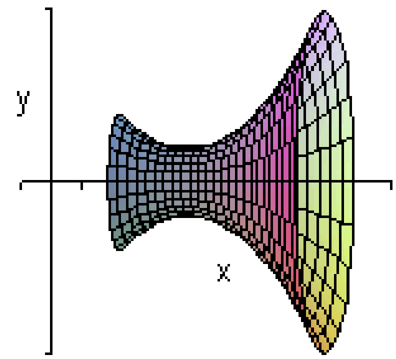
Day 6 Notes: Volume of Solids of Revolution

A solid of revolution is formed when a flat, two-dimensional shape is rotated around an axis. Consider the flat region below to the left. When that region is rotated about the x – axis, the solid pictured below to the right is formed. This objective of this lesson is to learn to find the volume of such a solid.





Now, imagine slicing the solid into individual discs of height 1 unit. The volume of one of those discs is

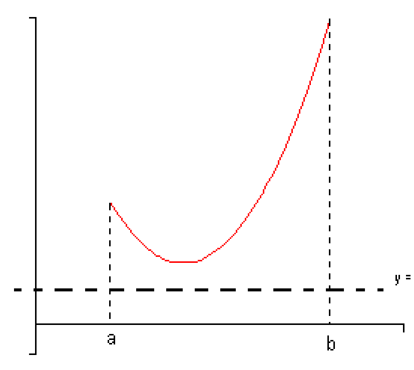
$V = \pi r^2 h$, or $V = \pi r^2$.

$$\text{Sum of all the discs} = \int_a^b \pi\left[f(x) - Axis\ of\ \ Rotation\right]^2 dx$$

$$\text{Volume of the Solid} = \pi\int_a^b \left[f(x) - Axis\ of\ \ Rotation\right]^2 dx$$

Notice that the axis of rotation is the x – axis and the bottom function of the region is also the x – axis. Imagine for a moment what the solid would look like if the axis of rotation were still the x – axis but the bottom function of the region was the line $y = c$. The solid would look similar except for the fact that there would be a cylinder that is cut out of the center.

To find the volume of this solid, we would find the volume of the whole solid that we found previously and then subtract out the solid in the form of a cylinder.



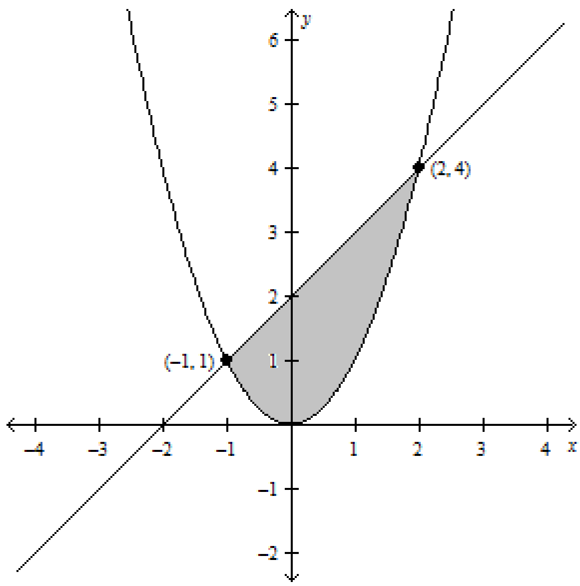
In order to do this, we use the formula below to find the volume of such a solid.

$$\text{Volume} = \pi\int_a^b [OuterFunction - axis]^2 - [InnerFunction - axis]^2 dx$$

The "outer function" is defined to be the function that is farther from the axis of rotation. The "inner function" is defined to be the function that is closer to the axis of rotation.

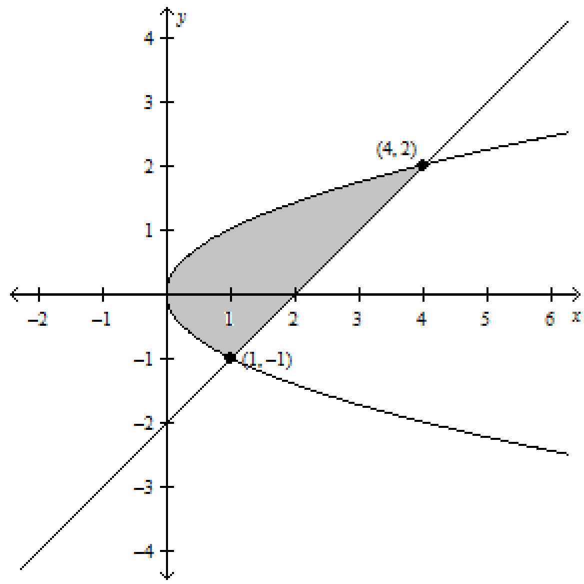
If the axis of rotation is the x-axis or is parallel to the x-axis, the integrand needs to be in terms of x and the limits of integration need to be the x-values of the points of intersection of the curves that form the region being rotated.

Consider the region pictured to the right that is bounded by the graphs of $y = x^2$ and $y = x + 2$.

Find the volume of the solid formed when the region is rotated about the x – axis.	
Find the volume when the region is rotated about the line $y = 4$.	Find the volume when the region is rotated about the line $y = –2$.

If the axis of rotation is the y-axis or is parallel to the y-axis, the integrand needs to be in terms of y and the limits of integration need to be the y-values of the points of intersection of the curves that form the region being rotated.

Consider the region pictured to the right that is bounded by the graphs $y = \pm\sqrt{x}$ and $y = x - 2$.

<table>
<tr>
<td>Find the volume when the region is rotated about the y – axis.</td>
<td>
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<tr>
<td>Find the volume when the region is rotated about the line $x = 4$.</td>
<td>Find the volume when the region is rotated about the line $x = 7$.</td>
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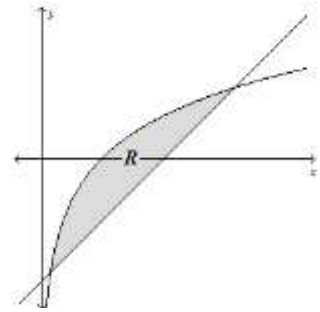
</table>

AP Calculus AB **Name: ______________________________**
Unit 7 – Day 6 – Assignment

Let R be the region bounded by the graphs of $y = \ln x$ and the line $y = x - 2$ as shown below. Though you may use a calculator, show the integral that you found to arrive at your answer.

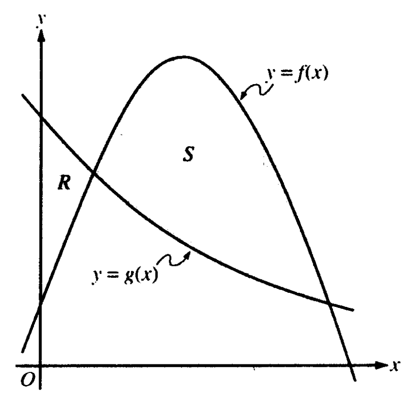


1. Find the coordinates of the points at which the two graphs intersect each other. Then, find the area of R.

2. Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

3. Write and evaluate an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the region in the first quadrant enclosed by the graphs of f and g shown to the right. Though you may use a calculator, show the integral that you found to arrive at your answer.



4. Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

5. Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.