AP Calculus
Unit 7 – Advanced Integration & Applications

Day 5 Notes: Finding the Area between Two Curves

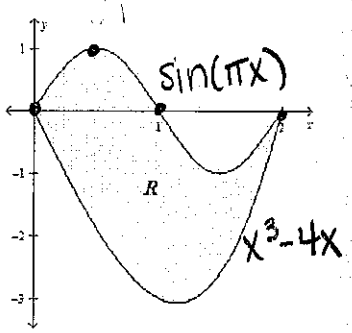
AREA BETWEEN TWO CURVES:

Area $= \int_a^b f(x) - g(x)\,dx$, provided that $f(x) > g(x)$ for $a < x < b$.

Example 1: Find the area of the shaded region, R, that is bounded by $y = \sin(\pi x)$ and $y = x^3 - 4x$.

math9

$$\int_0^2 \sin(\pi x) - (x^3 - 4x)\,dx = 4$$



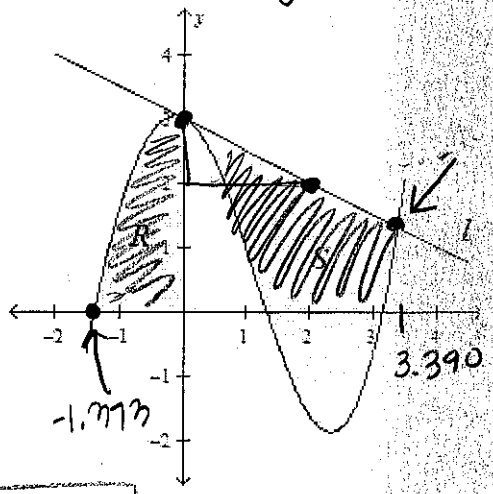
Example 2: Pictured to the right is the graph of $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$ and a line, l, which is tangent to $f(x)$ at the point (0, 3).

y_1 $y_2 = 0$

Find the area of Region R.

math9

$$\int_{-1.373}^{0} \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x\,dx = 2.903$$



Find the equation of line l if it is tangent to the graph of $f(x)$ at (0, 3). point slope $= -\frac{1}{2}$

$$y - 3 = -\frac{1}{2}(x - 0) \rightarrow y - 3 = -\frac{1}{2}x \quad y = 3 - \frac{1}{2}x$$

At what ordered pair, other than (0, 3), does the graph of line l intersect the graph of $f(x)$?

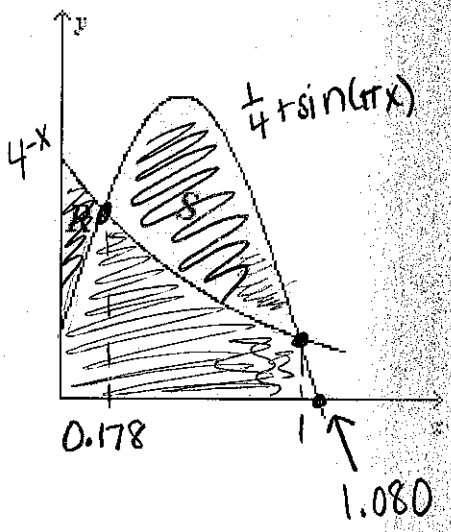
put $y_2 = 3 - \frac{1}{2}x$ Find intersection. (3.390, 1.305)

Find the area of Region S.

math9

$$\int_0^{3.390} \left(3 - \frac{1}{2}x\right) - \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x^2}{2} + 3\cos x\right)dx = 6.982$$

Example 3: Pictured to the right are regions R and S, which are formed by the graphs of $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$



Identify the points of intersection of $f(x)$ and $g(x)$.

(0.178, 0.781)

(1, 0.25)

Find the area of Region R.

math 9

$$\int_0^{0.178} 4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right) dx = 0.065$$

Find the area of Region S.

math 9

$$\int_{0.178}^{1} \frac{1}{4} + \sin(\pi x) - 4^{-x}\, dx = 0.410$$

Find the area of the unshaded region bounded by the graphs of f, g, and the x – axis.

math 9

$$\int_0^{1.080} \frac{1}{4} + \sin(\pi x) - \int_{.0178}^{1} \frac{1}{4} + \sin(\pi x) - 4^{-x}\, dx$$

$$= 0.897 - 0.410$$

$$= 0.487$$