Scavenger Hunt

A. If $y = x^2 e^x$, then $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = (2x)(e^x) + (x^2)(e^x)$$

$$= xe^x(2+x)$$

$$= \boxed{xe^x(x+2)}$$

I. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t=0$ to $t=2$?

$$\int_0^2 e^t\,dt = e^t\Big]_0^2 = e^2 - e^0$$

$$= \boxed{e^2 - 1}$$

E. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that...

$$f(x) = -5(x-2)^{-1}$$
$$f'(x) = 5(x-2)^{-2}(1)$$
$$f''(x) = -10(x-2)^{-3} = \frac{-10}{(x-2)^3} = 0$$

$$x = 2$$

$f''(x)$

1	2	3
$\frac{-10}{(1-2)^3}$		$\frac{-10}{(3-2)^3}$
$+$		$-$
conc. up		conc. down

$$\boxed{x > 2}$$

D. $\int \sec^2 x \, dx = \boxed{\tan x + C}$

(J.) If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} = ?$

$$\frac{dy}{dx} = \frac{(\cancel{x})(\frac{1}{\cancel{x}}) - (\ln x)(1)}{(x)^2}$$

$$= \boxed{\frac{1 - \ln x}{x^2}}$$

(B.) $\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

$6x$

$$\frac{1}{6}\int \underline{6\cancel{x}}\,(3x^2+5)^{-1/2}\,dx$$

$$\frac{1}{6}\left[\frac{(3x^2+5)^{1/2}}{1/2}\right] + C$$

$\frac{1}{6} \cdot \frac{2}{1} = \frac{2}{6}$ $\boxed{\frac{1}{3}(3x^2+5)^{1/2} + C}$

D. If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is ...

$$x + 2xy - y^2 = 2$$

$$1 + (2)(y) + (2x)(1)\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x - 2y) = -1 - 2y$$

$$\frac{dy}{dx} = \frac{-1-2y}{2x-2y}$$

$$= \frac{-1-2(1)}{2(1)-2(1)} = \frac{-3}{0}$$

$\boxed{\text{nonexistent}}$

F. If $\int_0^k (2kx - x^2)\,dx = 18$, then $k = ?$

$$\frac{\cancel{2}kx^2}{\cancel{2}} - \frac{1}{3}x^3\Big]_0^k = \left[k(k)^2 - \frac{1}{3}(k)^3\right] - \cancel{\left[k(0)^2 - \frac{1}{3}(0)^3\right]}$$

$$= k^3 - \frac{1}{3}k^3$$

$$k^3 - \frac{1}{3}k^3 = 18$$

$18 \cdot \frac{3}{2} = 9(3)$

$= 27$

$$\frac{2}{3}k^3 = 18$$

$$k^3 = 27$$

$\boxed{k = 3}$

K. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is...

$$f'(x) = (1)(1-2x)^3 + (x)\,3(1-2x)^2(-2)$$

$$f'(x) = (1-2x)^3 - 6x(1-2x)^2$$

$$f'(1) = (1-2(1))^3 - 6(1)(1-2(1))^2$$

$$= (-1)^3 - 6(-1)^2$$

$$= -1 - 6 = -7 \rightarrow \text{slope}$$

point $(1,-1)$

$$y+1 = -7(x-1)$$

$$y+1 = -7x + 7$$

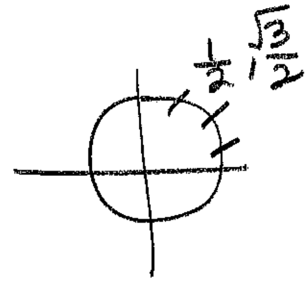
$$\boxed{y = -7x + 6}$$

N. If $f(x) = \sin x$, then $f'(\pi/3) = ?$

$$f'(x) = \cos x$$

$$f'(\pi/3) = \cos(\pi/3)$$

$$= \boxed{\frac{1}{2}}$$



G. If the function f has a continuous derivative on [0,c], then

$$\int_0^c f'(x)\,dx = ?$$

$$f(x)\Big]_0^c = \boxed{f(c) - f(0)}$$

L. $$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$$

$\cos\theta$

$$\int_0^{\pi/2} \cos\theta\,(1+\sin\theta)^{-1/2}\,d\theta$$

$$\frac{(1+\sin\theta)^{1/2}}{1/2}\Big]_0^{\pi/2}$$

$$2\sqrt{1+\sin\theta}\,\Big]_0^{\pi/2}$$

$$= 2\sqrt{1+\sin(\pi/2)} - 2\sqrt{1+\sin(0)}$$

$$= 2\sqrt{1+1} - 2\sqrt{1+0}$$

$$= 2\sqrt{2} - 2$$

$$= \boxed{2(\sqrt{2} - 1)}$$

C. If $f(x)=\sqrt{2x}$, then $f'(8)=?$

$f(x)=(2x)^{1/2}$

$f'(x)=\cancel{\frac{1}{2}}(2x)^{-1/2}(\cancel{2})$

$f'(x)=\frac{1}{\sqrt{2x}}$ $\quad f'(8)=\frac{1}{\sqrt{2(8)}}=\frac{1}{\sqrt{16}}=\boxed{\frac{1}{4}}$

M. If $y=2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2}=?$

$\frac{dy}{dx}=-2\sin\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)$

$=-\sin\left(\frac{1}{2}x\right)$

$\frac{d^2y}{dx^2}=-\cos\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)$

$=\boxed{-\frac{1}{2}\cos\left(\frac{x}{2}\right)}$

11.
$$\int_2^3 \frac{x}{x^2+1}\,dx$$

$$2x$$

$$= \frac{1}{2}\int_2^3 2x(x^2+1)^{-1}\,dx$$

$$= \frac{1}{2}\left[\frac{(x^2+1)^0}{0}\right]_2^3$$

$$= \frac{1}{2}\ln|x^2+1|\Big]_2^3$$

$$= \frac{1}{2}\ln|3^2+1| - \frac{1}{2}\ln|2^2+1|$$

$$= \frac{1}{2}\ln 10 - \frac{1}{2}\ln 5$$

$$= \frac{1}{2}\ln\frac{10}{5}$$

$$= \boxed{\frac{1}{2}\ln 2}$$