

AP Calculus

Unit 7 – Advanced Integration & Applications

Day 4 Notes: Slope Fields

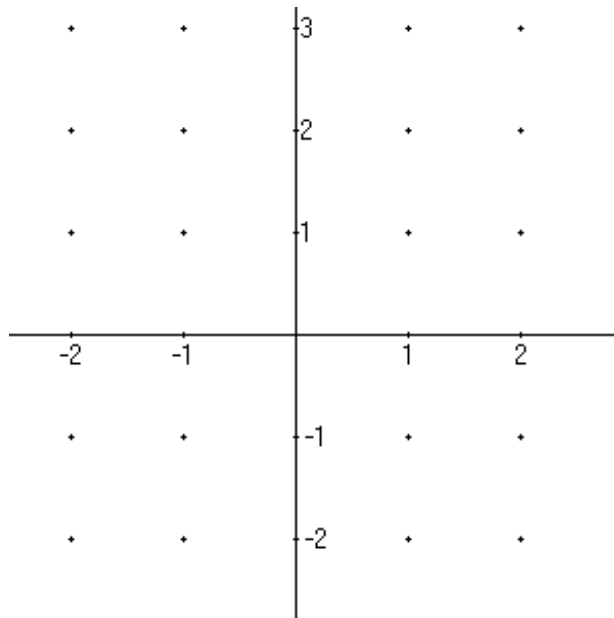
A **slope field** is a pictorial representation of all of the possible solutions to a given differential equation.

Remember that a differential equation is the first derivative of a function, $f'(x)$ or $\frac{dy}{dx}$. Thus, the solution to a differential equation is the function, $f(x)$ or y .

There is an infinite number of solutions to the differential equation $\frac{dy}{dx} = x - 1$. Show your work and explain why.

Given the differential equation below, compute the slope for each point. Indicated on the grid to the right. Then, make a small mark that approximates the slope through the point.

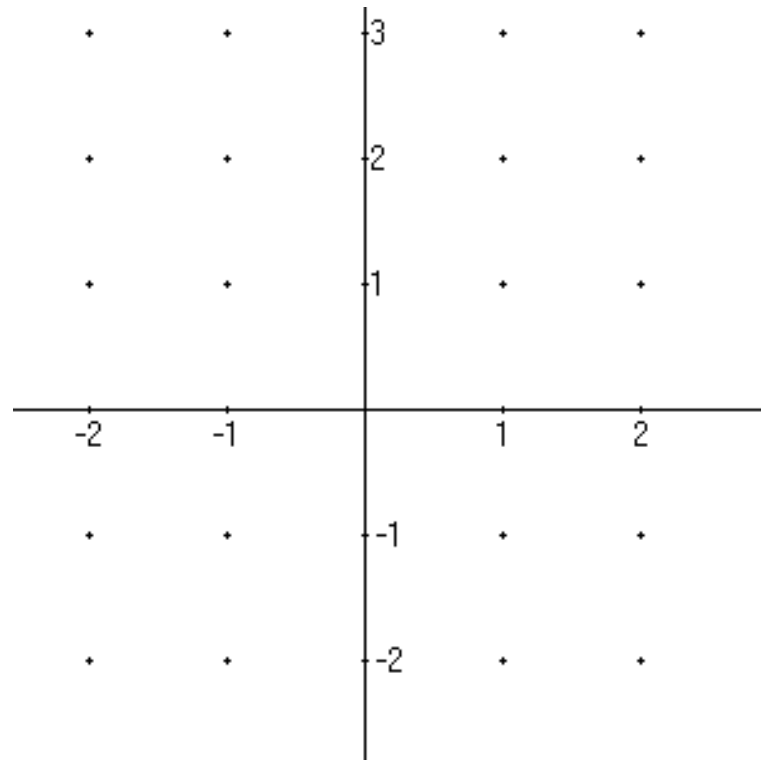
$$\frac{dy}{dx} = x - 1$$



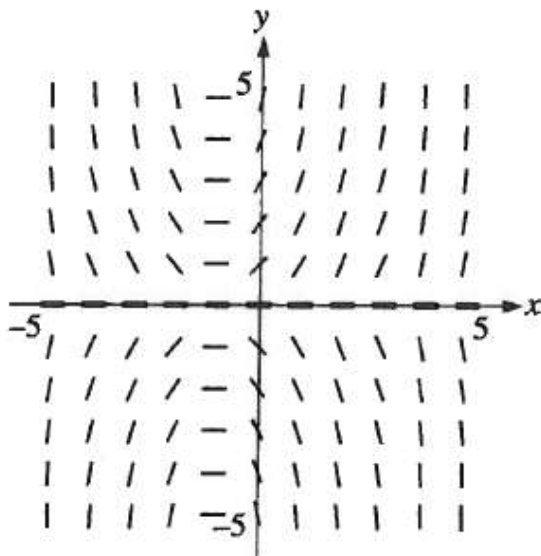
Notice how the segments drawn on the grid above would form parabolas if they were connected.

Given the differential equation below, compute the slope for each point indicated on the grid to the right. Then, make a small mark that approximates the slope through the point.

$$\frac{dy}{dx} = x - y$$



Shown below is a slope field for which of the following differential equations? Explain your reasoning for each of the choices below.

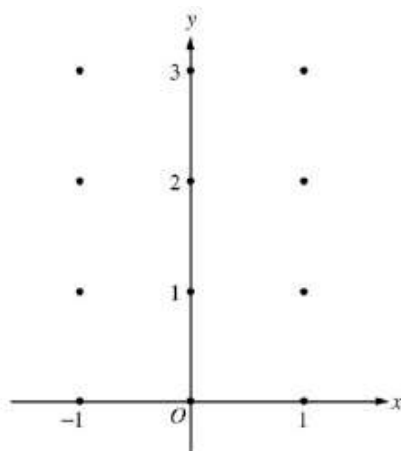


- (A) $\frac{dy}{dx} = xy$
- (B) $\frac{dy}{dx} = xy - y$
- (C) $\frac{dy}{dx} = xy + y$
- (D) $\frac{dy}{dx} = xy + x$
- (E) $\frac{dy}{dx} = (x + 1)^3$

2004 AP[®] CALCULUS AB
Question 5 (Form B)

Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



2008 AP[®] CALCULUS AB
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

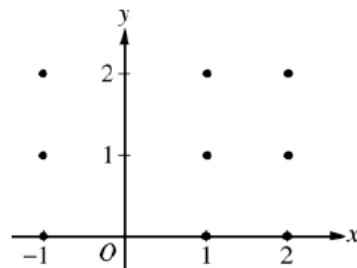
- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

- (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

- (c) For the particular solution $y = f(x)$ described in part (b), find

$$\lim_{x \rightarrow \infty} f(x).$$



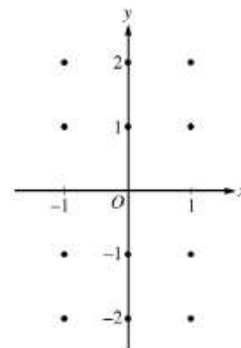
2010 AP[®] CALCULUS AB (Form B)
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)

- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane for which $y \neq 0$. Describe all points in the xy -plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.



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Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

- On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)
- While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are positive.
- Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

