AP Calculus AB
Unit 7 – Day 4 – Assignment

Name: Answer Key*

2010 AP® CALCULUS AB (Form B)
Question 5

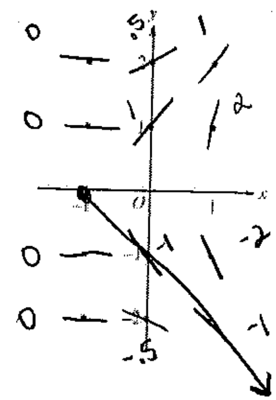
Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.



(a) $\frac{dy}{dx} = \frac{x+1}{y}$

slope eqn.

(b) $-1 = \frac{x+1}{y}$

$-y = x+1$

$y = -x-1$

$\boxed{\frac{dy}{dx} = -1 \text{ for all } (x,y) \text{ with } y=-x-1 \text{ \& } y \neq 0}$

(c) $\frac{dy}{dx} = \frac{x+1}{y}$

$\int y\,dy = \int (x+1)\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C$

$f(0) = -2$ (x, y)

$\frac{1}{2}(-2)^2 = \frac{1}{2}\cancel{(0)}^2 + \cancel{0} + C$

$C = 2$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + 2$

$y^2 = x^2 + 2x + 4$ $\qquad f(0) = \ominus 2$

$y = \pm\sqrt{x^2+2x+4}$

$\boxed{f(x) = -\sqrt{x^2+2x+4}}$

2004 AP® CALCULUS AB

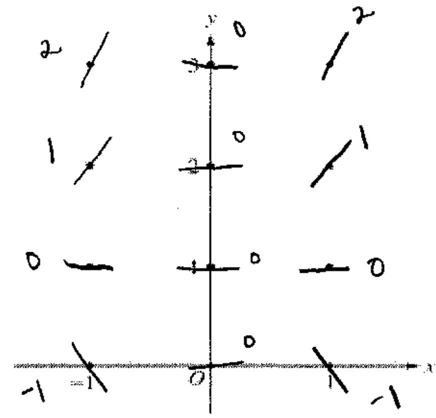
Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.



ⓐ $\frac{dy}{dx} = x^2(y-1)$

slope eqn

ⓑ $x^2(y-1)$

$\uparrow$ always ⊕ $\quad \uparrow$ $y>1$

$\boxed{\text{slopes are positive at points } (x,y) \text{ when } y>1 \text{ \& } x \neq 0}$

ⓒ $\frac{dy}{dx} = x^2(y-1)$

$\frac{dy}{y-1} = \frac{x^2(\cancel{y-1})\,dx}{\cancel{y-1}}$

$\int (y-1)^{-1}\,dy = \int x^2\,dx$

$\frac{(y-1)^0}{0} = \frac{1}{3}x^3 + C$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$f(0) = 3$

$\quad x \quad y$

$\ln|3-1| = \frac{1}{3}(\cancel{0})^3 + C$

$C = \ln 2$

$e^{\ln|y-1|} = e^{\frac{1}{3}x^3 + \ln 2}$

$|y-1| = e^{\frac{1}{3}x^3} \cdot \cancel{e}^{\ln 2}$

$|y-1| = 2e^{\frac{1}{3}x^3}$

$y-1 = 2e^{\frac{1}{3}x^3}$ or $y-1 = -2e^{\frac{1}{3}x^3}$

$y = 1 + 2e^{\frac{1}{3}x^3}$ ✓ $\quad$ $\cancel{y = 1 - 2e^{\frac{1}{3}x^3}}$

$f(0) = 3$

$y = 1 + 2e^{\frac{1}{3}(0)^3}$

$1 + 2 = 3$ ✓

$\boxed{f(x) = 1 + 2e^{\frac{1}{3}x^3}}$