AP Calculus
Unit 7 – Advanced Integration & Applications

Day 3 Notes: Solving Differential Equations

Given below are differential equations with given initial condition values. Find the particular solution, $y = f(x)$, for each differential equation that satisfies the given initial condition.

1. $\frac{dy}{dx} = 6x^2 + 6x + 2$ and $f(-1) = 2$	2. $\frac{dy}{dx} = \frac{1 + 12x^{3/2}}{2\sqrt{x}}$ and $f(0) = 2$
3. $\frac{dy}{dx} = \frac{x^2 + 2x}{2y}$ and $f(0) = 2$	4. $\frac{dy}{dx} = \frac{x + 2}{y}$ and $f(1) = -3$

5. $\frac{dy}{dx} = x^4(y-2)$ and $f(0) = 0$	6. $\frac{dy}{dx} = \frac{y-1}{x^2}$ and $f(2) = 0$

2000 AP Calculus AB
Question 6

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

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Unit 7 – Day 3 – Assignment

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The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

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Unit 7 – Day 4 – Warm-up

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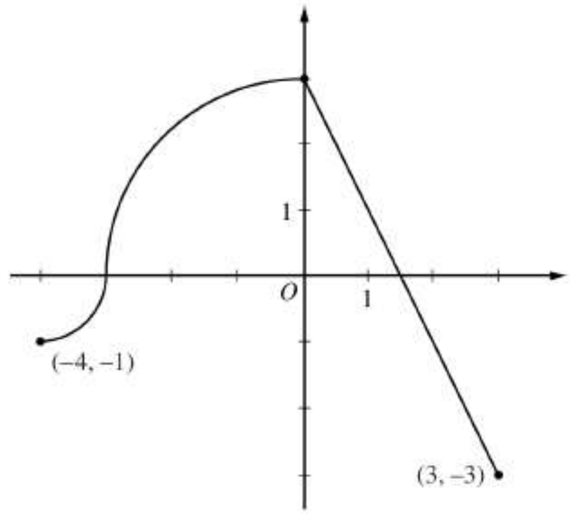
Question 4



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.