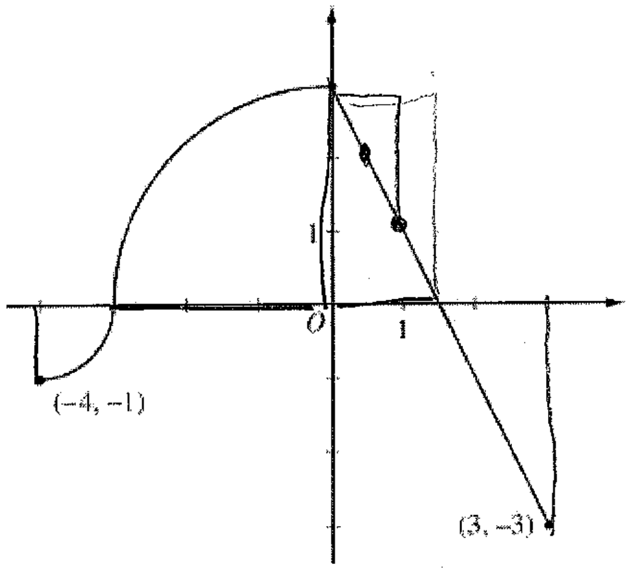
AP Calculus AB
Unit 7 – Day 2 – Warm-up

Name: Answer Key*



$f(t)$

Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph consists of two quarter circles and one line segment, as show in the figure above. Let $g(x) = \frac{1}{2}x^2 + \int_0^x f(t)dt$.

Find the value of $g(3)$. $g(3) = \frac{1}{2}(3)^2 + \underbrace{\int_0^3 f(t)dt}_{\text{area}}$ $= 4.5 + \frac{1}{2}(\cancel{1.5})(3) - \frac{1}{2}(\cancel{1.5})(3)$ $\boxed{g(3) = 4.5}$	Find the value of $g(-4)$. $g(-4) = \frac{1}{2}(-4)^2 + \int_0^{-4} f(t)dt$ $= 8 + -\underbrace{\int_{-4}^{0} f(t)dt}_{\text{area}}$ $= 8 + -\left[\frac{1}{4}\pi(3)^2 - \frac{1}{4}\pi(1)^2\right] = 8 - \frac{9}{4}\pi + \frac{1}{4}\pi$ $= \boxed{8 - 2\pi}$
Find the value of $g'(3)$. $g'(x) = x + f(x)(1)$ $g'(3) = (3) + f(3)$ $= 3 + -3$ $= \boxed{0}$	Find the value of $g''(2)$. $g'(x) = x + f(x)$ $g''(x) = 1 + f'(x)$ $g''(2) = 1 + \underbrace{f'(2)}$ slope of tangent $= 1 + -2$ $= \boxed{-1}$

AP Calculus
Unit 7 – Advanced Integration & Applications

Day 2 Notes: Integration of Composite Functions

Anti-differentiation by Pattern Recognition

ex. $f(x) = (2x+1)^3$
$f'(x) = 3(2x+1)^2(2)$

$\frac{d}{dx}[f(g(x))] =$ $f'(g(x)) \cdot g'(x)$

$\int f'(g(x)) \cdot g'(x)\,dx =$ $f(g(x)) + C$

Find each of the following indefinite integrals by pattern recognition.

$\int 2x\sqrt[3]{x^2+5}\,dx$ $2x$ $\int 2x(x^2+5)^{1/3}dx$ $\frac{(x^2+5)^{4/3}}{4/3} + C$ $\boxed{\frac{3}{4}(x^2+5)^{4/3} + C}$	$\int 3\cos 3x\,dx$ 3 $\boxed{\sin(3x) + C}$
$\int 3x\sqrt{x^2+2}\,dx$ $2x$ $\frac{1}{2} \cdot 3\int 2x(x^2+2)^{1/2}dx$ $\frac{3}{2}\left[\frac{(x^2+2)^{3/2}}{3/2}\right] + C$ $\boxed{(x^2+2)^{3/2} + C}$	$2x+2$ $\int \frac{2x+2}{x^2+2x}dx = \int (2x+2)(x^2+2x)^{-1}dx$ $\frac{(x^2+2x)^0}{0} + C$ $\boxed{\ln\lvert x^2+2x\rvert + C}$
$\frac{1}{3}\int 3\cos(3x+2)\,dx$ 3 $\frac{1}{3}(\sin(3x+2)) + C$ $\boxed{\frac{1}{3}\sin(3x+2) + C}$	$\int 2\sin(2x+3)\,dx$ 2 $\boxed{-\cos(2x+3) + C}$
$\frac{1}{3} \cdot 5\int 3e^{3x}\,dx$ 3 $\frac{5}{3}[e^{3x}] + C$ $\boxed{\frac{5}{3}e^{3x} + C}$	$4x$ $\int \frac{3x}{\sqrt{2x^2+3}}dx$ $\frac{1}{4} \cdot 3\int 4x(2x^2+3)^{-1/2}dx$ $\frac{3}{4}\left[\frac{(2x^2+3)^{1/2}}{1/2}\right] + C$

$\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}}$

$\frac{3}{4} \cdot \frac{2}{1} = \frac{6}{4}$

$\boxed{\frac{3}{2}(2x^2+3)^{1/2} + C}$

Anti-differentiation by U–Substitution

In each of the eight examples above, the $g'(x)$ existed in the integrand of $\int f'(g(x)) \cdot g'(x)\,dx$ or $g'(x)$ was attainable by multiplying by a constant. The $g'(x)$ does not always exist and there are times when it is not attainable by multiplication of a constant. Consider the example below.

$$\int x(2x-1)^3 dx$$

Identify the "inner function," $g(x)$: $2x-1$

What is $g'(x)$? 2 Is $g'(x)$ part of the integrand? no

Is $g'(x)$ attainable by multiplying the integrand by a constant? no

In this case, we must find the anti-derivative by a method known as U-Substitution. Here is how it works.

1. Let u = the inner function, $g(x)$. $u = 2x-1$	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of u. Then, anti-differentiate. $\int x(2x-1)^3 dx$ $\int \left(\frac{u+1}{2}\right)(u)^3\left(\frac{du}{2}\right)$ $\int \frac{1}{4}(u+1)(u^3)\,du$ $\frac{1}{4}\int u^4+u^3\,du$ $\frac{1}{4}\left[\frac{u^5}{5}+\frac{u^4}{4}\right]+C$ $\frac{1}{20}u^5+\frac{1}{16}u^4+C$ $\boxed{\frac{1}{20}(2x-1)^5+\frac{1}{16}(2x-1)^4+C}$
2. Find du and solve the equation for dx. $du = 2dx$ $\frac{du}{2} = dx$	
3. Find an expression for x in terms of u. $u=2x-1$ $\frac{u+1}{2} = x$	

$$\int \frac{2x+1}{\sqrt{x+4}}\,dx = \int (2x+1)(x+4)^{-1/2}\,dx$$

1. Let u = the inner function, $g(x)$.

$$u = x+4$$

2. Find du and solve the equation for dx.

$$du = 1\,dx$$

$$dx = du$$

3. Find an expression for x in terms of u.

$$u = x+4$$

$$u-4 = x$$

4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of u. Then, anti-differentiate.

$$\int (2x+1)(x+4)^{-1/2}\,dx$$

$$\int (2(u-4)+1)(u)^{-1/2}\,du$$

$$\int (2u-8+1)(u^{-1/2})\,du$$

$$\int (2u-7)(u^{-1/2})\,du$$

$$\int 2u^{1/2} - 7u^{-1/2}\,du$$

$$\frac{2u^{3/2}}{3/2} - \frac{7u^{1/2}}{1/2} + C$$

$2 \cdot \frac{2}{3}$ $\quad \frac{4}{3}u^{3/2} - 14u^{1/2} + C$

$7 \cdot \frac{2}{1}$ $\quad \boxed{\frac{4}{3}(x+4)^{3/2} - 14(x+4)^{1/2} + C}$