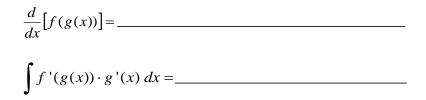
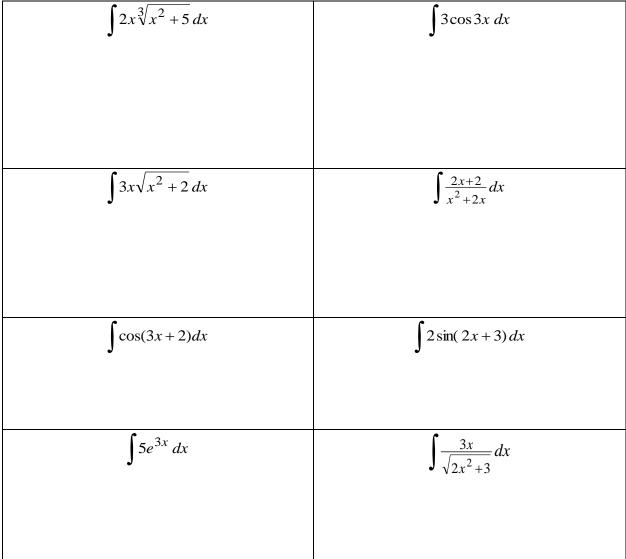
AP Calculus Unit 7 – Advanced Integration & Applications

Day 2 Notes: Integration of Composite Functions

Anti-differentiation by Pattern Recognition



Find each of the following indefinite integrals by pattern recognition.



Anti-differentiation by U–Substitution

In each of the eight examples above, the g'(x) existed in the integrand of $\int f'(g(x)) \cdot g'(x) dx$ or g'(x) was attainable by multiplying by a constant. The g'(x) does not always exist and there are times when it is not attainable by multiplication of a constant. Consider the example below.

$$\int x(2x-1)^3 dx$$

 Identify the "inner function," g(x):

 What is g'(x)?

 Is g'(x) part of the integrand?

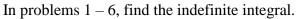
 Is g'(x) attainable by multiplying the integrand by a constant?

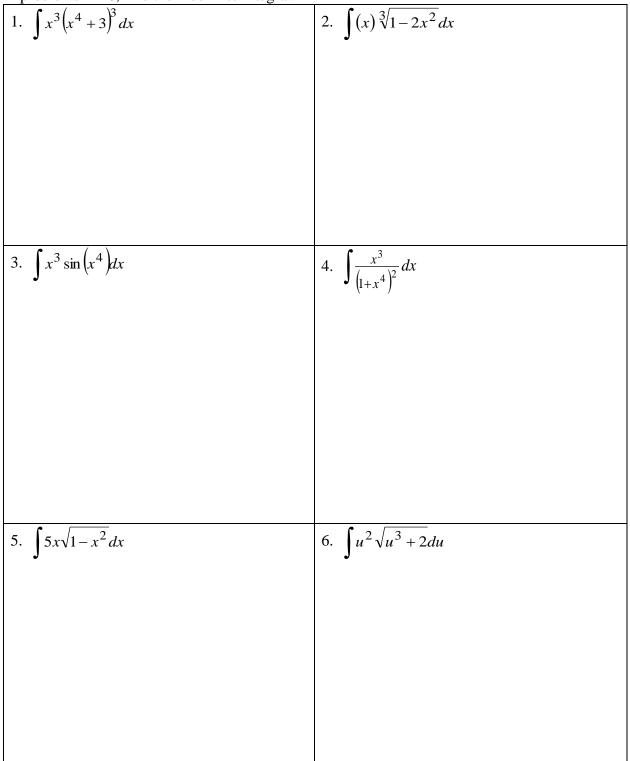
In this case, we must find the anti-derivative by a method known as U-Substitution. Here is how it works.

1. Let $u =$ the inner function, $g(x)$.	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of <i>u</i> . Then, anti-differentiate.
2. Find <i>du</i> and solve the equation for <i>dx</i> .	
3. Find an expression for <i>x</i> in terms of <i>u</i> .	

$\int \frac{2x+1}{\sqrt{x+4}} dx$	
1. Let $u =$ the inner function, $g(x)$.	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of <i>u</i> . Then, anti-differentiate.
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Name: _____





7.
$$\int \frac{2x^2}{\sqrt{x^3 - 2}} dx =$$

A. $\frac{4}{3} (x^3 - 2)^{\frac{1}{2}} + C$
B. $\frac{1}{3} (x^3 - 2)^{\frac{1}{2}} + C$
C. $\frac{2}{3} (x^3 - 2)^{\frac{1}{2}} + C$
D. $2 (x^3 - 2)^{\frac{1}{2}} + C$
E. $3 (x^3 - 2)^{\frac{1}{2}} + C$

8. If
$$u = 2x - 3$$
, then $\int x \sqrt[3]{2x - 3} dx =$
A. $\frac{1}{4} \int u^{\frac{4}{3}} + 3u^{\frac{1}{3}} du$
B. $\frac{1}{2} \int u^{\frac{2}{3}} + 3u^{\frac{4}{3}} du$
C. $\frac{1}{2} \int \sqrt[3]{u} du$
D. $\frac{1}{4} \int \sqrt[3]{u} du$
E. $\frac{1}{2} \int (2u + 3)^{\frac{3}{4}u} du$

9. If $\frac{dy}{dx} = \frac{x^2}{y}$ and f(0) = -4, find the particular solution to the differential equation.

A.
$$f(x) = \frac{1}{3}x^3 + 4$$

B. $f(x) = -\sqrt{\frac{2}{3}x^3 + 16}$
C. $f(x) = \sqrt{\frac{2}{3}x^3 + 16}$
D. $f(x) = \frac{1}{3}x^3$
E. $f(x) = -\sqrt{\frac{2}{3}x^3 + 8}$

10. Using the substitution $u = \sqrt{x}$, $\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to which of the following?

A.
$$2\int_{1}^{16} e^{u} du$$
 B. $2\int_{1}^{4} e^{u} du$

C.
$$2\int_{1}^{2} e^{u} du$$
 D. $\frac{1}{2}\int_{1}^{2} e^{u} du$

E.
$$\int_1^4 e^u du$$

11. $\int_{0}^{1} e^{-4x} dx =$ A. $-\frac{e^{-4}}{4}$ B. $-4e^{-4}$ C. $e^{-4} - 1$ D. $\frac{1}{4} - \frac{e^{-4}}{4}$ E. $4 - 4e^{-4}$

12.
$$\int \frac{x}{x^2 - 4} dx =$$

A.
$$\frac{-1}{4(x^2 - 4)^2} + C$$

B.
$$\frac{1}{2(x^2 - 4)} + C$$

C.
$$\frac{1}{2} \ln |x^2 - 4| + C$$

D.
$$2 \ln |x^2 - 4| + C$$

E.
$$\frac{2}{x^2 - 4} + C$$