AP Calculus
Unit 7 – Advanced Integration & Applications

Day 2 Notes: Integration of Composite Functions

Anti-differentiation by Pattern Recognition

$$\frac{d}{dx}\left[f(g(x))\right] = \underline{\hspace{8cm}}$$

$$\int f'(g(x)) \cdot g'(x)\, dx = \underline{\hspace{8cm}}$$

Find each of the following indefinite integrals by pattern recognition.

$\int 2x\sqrt[3]{x^2+5}\, dx$	$\int 3\cos 3x\, dx$
$\int 3x\sqrt{x^2+2}\, dx$	$\int \frac{2x+2}{x^2+2x}\, dx$
$\int \cos(3x+2)dx$	$\int 2\sin(2x+3)\, dx$
$\int 5e^{3x}\, dx$	$\int \frac{3x}{\sqrt{2x^2+3}}\, dx$

Anti-differentiation by U–Substitution

In each of the eight examples above, the $g'(x)$ existed in the integrand of $\int f'(g(x)) \cdot g'(x)\, dx$ or $g'(x)$ was attainable by multiplying by a constant. The $g'(x)$ does not always exist and there are times when it is not attainable by multiplication of a constant. Consider the example below.

$$\int x(2x-1)^3\, dx$$

Identify the "inner function," $g(x)$: __________________________
What is $g'(x)$? ____________________ Is $g'(x)$ part of the integrand? ______________
Is $g'(x)$ attainable by multiplying the integrand by a constant? _________________________

In this case, we must find the anti-derivative by a method known as U-Substitution. Here is how it works.

1. Let u = the inner function, $g(x)$.	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of u. Then, anti-differentiate.
2. Find du and solve the equation for dx.	
3. Find an expression for x in terms of u.	

$$\int \frac{2x+1}{\sqrt{x+4}}\,dx$$

1. Let u = the inner function, $g(x)$.	4. Rewrite the entire integrand as a polynomial or polynomial type of function in terms of u. Then, anti-differentiate.
2. Find du and solve the equation for dx.	
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AP Calculus AB **Name: ______________________________**
Unit 7 – Day 2 – Assignment

In problems 1 – 6, find the indefinite integral.

1. $\int x^3\left(x^4+3\right)^3 dx$	2. $\int (x)\sqrt[3]{1-2x^2}\,dx$
3. $\int x^3 \sin\left(x^4\right) dx$	4. $\int \frac{x^3}{\left(1+x^4\right)^2} dx$
5. $\int 5x\sqrt{1-x^2}\,dx$	6. $\int u^2\sqrt{u^3+2}\,du$

7. $\int \frac{2x^2}{\sqrt{x^3-2}}\,dx =$

A. $\frac{4}{3}\left(x^3-2\right)^{1/2}+C$

B. $\frac{1}{3}\left(x^3-2\right)^{1/2}+C$

C. $\frac{2}{3}\left(x^3-2\right)^{1/2}+C$

D. $2\left(x^3-2\right)^{1/2}+C$

E. $3\left(x^3-2\right)^{1/2}+C$

8. If $u = 2x - 3$, then $\int x\sqrt[3]{2x-3}\,dx =$

A. $\frac{1}{4}\int u^{4/3}+3u^{1/3}\ du$

B. $\frac{1}{2}\int u^{2/3}+3u^{4/3}\ du$

C. $\frac{1}{2}\int \sqrt[3]{u}\ du$

D. $\frac{1}{4}\int \sqrt[3]{u}\ du$

E. $\frac{1}{2}\int (2u+3)\sqrt[3]{u}\ du$

9. If $\frac{dy}{dx}=\frac{x^2}{y}$ and $f(0) = -4$, find the particular solution to the differential equation.

A. $f(x) = \frac{1}{3}x^3+4$

B. $f(x) = -\sqrt{\frac{2}{3}x^3+16}$

C. $f(x) = \sqrt{\frac{2}{3}x^3+16}$

D. $f(x) = \frac{1}{3}x^3$

E. $f(x) = -\sqrt{\frac{2}{3}x^3+8}$

10. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

A. $2\int_1^{16} e^u\,du$

B. $2\int_1^{4} e^u\,du$

C. $2\int_1^{2} e^u\,du$

D. $\frac{1}{2}\int_1^{2} e^u\,du$

E. $\int_1^{4} e^u\,du$

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11. $\int_0^1 e^{-4x}\,dx =$

A. $-\frac{e^{-4}}{4}$

B. $-4e^{-4}$

C. $e^{-4} - 1$

D. $\frac{1}{4} - \frac{e^{-4}}{4}$

E. $4 - 4e^{-4}$

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12. $\int \frac{x}{x^2 - 4}\,dx =$

A. $\frac{-1}{4\left(x^2-4\right)^2} + C$

B. $\frac{1}{2\left(x^2-4\right)} + C$

C. $\frac{1}{2}\ln\left|x^2 - 4\right| + C$

D. $2\ln\left|x^2 - 4\right| + C$

E. $\frac{2}{x^2 - 4} + C$