AP Calculus AB
Unit 7 – Day 2 – Assignment

Name: Answer Key*

In problems 1 – 6, find the indefinite integral.

1. $\frac{1}{4}\int 4x^3(x^4+3)^3\,dx$ $4x^3$

$\frac{1}{4}\left[\frac{(x^4+3)^4}{4}\right]+C$

$\frac{1}{4}\cdot\frac{1}{4}$

$\boxed{\frac{1}{16}(x^4+3)^4+C}$

$-4x$

2. $\int (x)\sqrt[3]{1-2x^2}\,dx = -\frac{1}{4}\int -4x\,(1-2x^2)^{1/3}\,dx$

$-\frac{1}{4}\left[\frac{(1-2x^2)^{4/3}}{4/3}\right]+C$

$-\frac{1}{4}\cdot\frac{3}{4}$

$\boxed{-\frac{3}{16}(1-2x^2)^{4/3}+C}$

3. $\frac{1}{4}\int 4x^3\sin(x^4)\,dx$ $4x^3$

$\frac{1}{4}\left[-\cos(x^4)\right]+C$

$\boxed{-\frac{1}{4}\cos(x^4)+C}$

4. $\int \frac{x^3}{(1+x^4)^2}\,dx = \frac{1}{4}\int 4x^3(1+x^4)^{-2}\,dx$ $4x^3$

$\frac{1}{4}\left[\frac{(1+x^4)^{-1}}{-1}\right]+C$

$\boxed{-\frac{1}{4(1+x^4)}+C}$

$-2x$

5. $\int 5x\sqrt{1-x^2}\,dx = -\frac{1}{2}\cdot 5\int -2x\,(1-x^2)^{1/2}\,dx$

$-\frac{5}{2}\left[\frac{(1-x^2)^{3/2}}{\frac{3}{2}}\right]+C$

$-\frac{5}{\cancel{2}}\cdot\frac{\cancel{2}}{3}$

$\boxed{-\frac{5}{3}(1-x^2)^{3/2}+C}$

$3u^2$

6. $\int u^2\sqrt{u^3+2}\,du = \frac{1}{3}\int 3u^2(u^3+2)^{1/2}\,du$

$\frac{1}{3}\left[\frac{(u^3+2)^{3/2}}{3/2}\right]+C$

$\frac{1}{3}\cdot\frac{2}{3}$

$\boxed{\frac{2}{9}(u^3+2)^{3/2}+C}$

$3x^2$

7. $\int \frac{2x^2}{\sqrt{x^3-2}}dx =$ $\frac{1}{3}\cdot 2\int \frac{3}{2}x^2(x^3-2)^{-1/2}dx$

A. $\frac{4}{3}\left(x^3-2\right)^{1/2}+C$

B. $\frac{1}{3}\left(x^3-2\right)^{1/2}+C$

C. $\frac{2}{3}\left(x^3-2\right)^{1/2}+C$

D. $2\left(x^3-2\right)^{1/2}+C$

E. $3\left(x^3-2\right)^{1/2}+C$

$\frac{2}{3}\left[\frac{(x^3-2)^{1/2}}{1/2}\right]+C$

$\frac{2}{3}\cdot\frac{2}{1}$ $\quad \frac{4}{3}(x^3-2)^{1/2}+C$

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8. If $u = 2x - 3$, then $\int x\sqrt[3]{2x-3}\,dx =$

$u=2x-3$

$du=2dx$

$\frac{du}{2}=dx$

$x=\frac{u+3}{2}$

A. $\frac{1}{4}\int u^{4/3}+3u^{1/3}\ du$

B. $\frac{1}{2}\int u^{2/3}+3u^{4/3}\ du$

C. $\frac{1}{2}\int \sqrt[3]{u}\ du$

D. $\frac{1}{4}\int \sqrt[3]{u}\ du$

E. $\frac{1}{2}\int (2u+3)\sqrt[3]{u}\ du$

$\int x\,(2x-3)^{1/3}\,dx$

$\int\left(\frac{u+3}{2}\right)(u)^{1/3}\left(\frac{du}{2}\right)$

$\int \frac{1}{4}(u+3)(u)^{1/3}$

$\frac{1}{4}\int u^{4/3}+3u^{1/3}$

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9. If $\frac{dy}{dx} = \frac{x^2}{y}$ and $f(0) = -4$, find the particular solution to the differential equation.

$x \quad y$

A. $f(x) = \frac{1}{3}x^3+4$

B. $f(x) = -\sqrt{\frac{2}{3}x^3+16}$

C. $f(x) = \sqrt{\frac{2}{3}x^3+16}$

D. $f(x) = \frac{1}{3}x^3$

E. $f(x) = -\sqrt{\frac{2}{3}x^3+8}$

$\int y\,dy = \int x^2\,dx$

$\frac{y^2}{2} = \frac{x^3}{3}+C$

plug in $x=0$, $y=-4$

$\frac{(-4)^2}{2} = \cancel{\frac{(0)^3}{3}}+C$

$8=C$

$\frac{y^2}{2}=\frac{x^3}{3}+8$

$y^2=\frac{2}{3}x^3+16 \rightarrow y=\pm\sqrt{\frac{2}{3}x^3+16}$

$f(0)=-4$

$y=(-)\sqrt{\frac{2}{3}(0)^3+16}$

$y=-4$

must be negative

you must $\sqrt{\ }$ the bounds b/c $u=\sqrt{x}$.

$u^2 = x$

$u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx \rightarrow \frac{du}{\frac{1}{2}x^{-1/2}} = dx \rightarrow$

$2x^{1/2}du = dx$

10. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

A. $2\int_1^{16} e^u\,du$

B. $2\int_1^{4} e^u\,du$

C. $2\int_1^{2} e^u\,du$

D. $\frac{1}{2}\int_1^{2} e^u\,du$

E. $\int_1^{4} e^u\,du$

$\int_{\sqrt{1}}^{\sqrt{4}} \frac{e^u}{u}(2x^{1/2}du)$

$\int_1^2 \frac{e^u}{\cancel{u}}(2\cancel{u}du)$

$2\int_1^2 e^u du$

11. $\int_0^1 e^{-4x}\,dx =$

-4

$-\frac{1}{4}\int_0^1 -4e^{-4x}$

A. $-\frac{e^{-4}}{4}$

B. $-4e^{-4}$

C. $e^{-4} - 1$

D. $\frac{1}{4} - \frac{e^{-4}}{4}$

E. $4 - 4e^{-4}$

$-\frac{1}{4}\left[e^{-4x}\right] \cancel{+C}\Big]_0^1$

$-\frac{1}{4}e^{-4(1)} - \left(-\frac{1}{4}e^{-4(0)}\right)$

$-\frac{1}{4}e^{-4} + \frac{1}{4}(1)$

$-\frac{1}{4}e^{-4} + \frac{1}{4}$

12. $\int \frac{x}{x^2-4}\,dx =$

$\frac{1}{2}\int 2x((x^2-4)^{-1})dx$

$2x$

$\frac{1}{2}\left[\frac{(x^2-4)^0}{0}\right] + C$

$\frac{1}{2}\ln|x^2-4| + C$

A. $\frac{-1}{4(x^2-4)^2} + C$

B. $\frac{1}{2(x^2-4)} + C$

C. $\frac{1}{2}\ln|x^2-4| + C$

D. $2\ln|x^2-4| + C$

E. $\frac{2}{x^2-4} + C$