AP Calculus
Unit 7 – Advanced Integration & Applications

Day 1 Notes: Second Fundamental Theorem of Calculus

derivative

Given the functions, $f(t)$, below, use $F(x) = \int_1^x f(t)dt$ to find $\boxed{F(x)}$ and $F'(x)$ in terms of x.

1. $f(t) = 4t - t^2$	2. $f(t) = \cos t$
$F(x) = \int_1^x 4t - t^2\,dt$ $\frac{4t^2}{2} - \frac{t^3}{3} + C\Big]_1^x$ $2t^2 - \frac{1}{3}t^3 + C\Big]_1^x$ $[2x^2 - \frac{1}{3}x^3] - [2(1)^2 - \frac{1}{3}(1)^3]$ $= 2x^2 - \frac{1}{3}x^3 - 2 + \frac{1}{3}$ $\boxed{F(x) = 2x^2 - \frac{1}{3}x^3 - \frac{5}{3}}$ $\boxed{F'(x) = 4x - x^2}$	$F(x) = \int_1^x \cos t\,dt$ $= \sin t\Big]_1^x$ $= \sin x - \sin 1$ $\boxed{F(x) = \sin x - \sin 1}$ $\boxed{F'(x) = \cos x}$

Given the functions, $f(t)$, below, use $F(x) = \int_1^{x^2} f(t)dt$ to find $F(x)$ and $F'(x)$ in terms of x.

3. $f(t) = t^3$	4. $f(t) = 6\sqrt{t}$
$F(x) = \int_1^{x^2} t^3\,dt$ $\frac{1}{4}t^4 + C\Big]_1^{x^2}$ $\frac{1}{4}(x^2)^4 - \frac{1}{4}(1)^4$ $\boxed{F(x) = \frac{1}{4}x^8 - \frac{1}{4}}$ $\boxed{F'(x) = 2x^7}$	$F(x) = \int_1^{x^2} 6t^{1/2}\,dt$ $\frac{6t^{3/2}}{3/2} + C\Big]_1^{x^2}$ $4t^{3/2} + C\Big]_1^{x^2}$ $4(x^2)^{3/2} - 4(1)^{3/2}$ $\boxed{F(x) = 4x^3 - 4}$ $\boxed{F'(x) = 12x^2}$

$6 \cdot \frac{2}{3} = \frac{12}{3} = 4$

$2 \cdot \frac{3}{3}$

Second Fundamental Theorem of Calculus

If $F(x)=\int_a^{g(x)} f(t)\,dt$, then
$F'(x)=f(g(x))\cdot g'(x)$

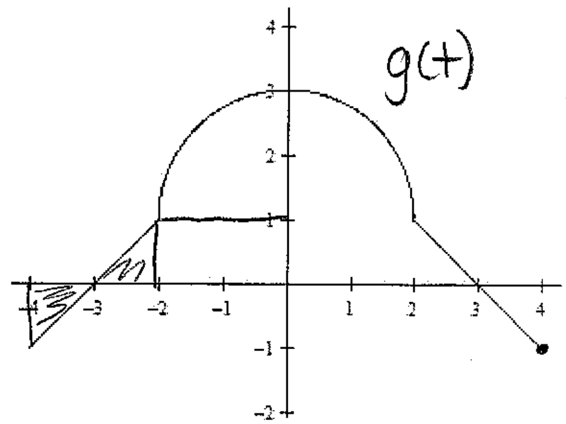
Complete the table below for each function.

Function	Find $F'(x)$ by applying the Second Fundamental Theorem of Calculus
$F(x)=\int_1^x \left(4t-t^2\right)dt$	$F(x)=\int_1^{x}\left(4t-t^2\right)dt$ $F'(x)=\left(4x-x^2\right)\cdot(1)$ $\boxed{F'(x)=4x-x^2}$ ✓
$F(x)=\int_1^x (\cos t)dt$	$F(x)=\int_1^{x}(\cos t)dt$ $F'(x)=(\cos x)(1)$ $\boxed{F'(x)=\cos x}$ ✓
$F(x)=\int_1^{x^2} t^3\,dt$	$F(x)=\int_1^{x^2} t^3\,dt$ $F'(x)=\left((x^2)^3\right)(2x)$ $\boxed{F'(x)=2x^7}$ ✓
$F(x)=\int_1^{x^2} 6\sqrt{t}\,dt$	$F(x)=\int_1^{x^2} 6\sqrt{t}\,dt$ $F'(x)=\left(6\sqrt{x^2}\right)(2x)$ $\boxed{F'(x)=12x^2}$ ✓

Find the derivative of each of the following functions.

$F(x)=\int_{-2}^{2x}\sqrt{2-t^2}\,dt$ $F'(x)=(\sqrt{2-(2x)^2})(2)$ $\boxed{F'(x)=2\sqrt{2-4x^2}}$	$G(x)=\int_{x^2}^{-3} e^{\cos t}\,dt$ $G(x)=-\int_{-3}^{x^2} e^{\cos t}\,dt$ $G'(x)=-(e^{\cos x^2})(2x)$ $\boxed{G'(x)=-2xe^{\cos x^2}}$	$H(x)=\int_{0}^{\cos x} t^2\,dt$ $H'(x)=((\cos x)^2)(-\sin x)$ $\boxed{H'(x)=-\cos^2 x\cdot\sin x}$

Pictured to the right is the graph of $g(t)$ and the function $f(x)$ is defined to be $f(x)=\int_{-4}^{2x} g(t)\,dt$.



1. Find the value of $f(0)$.

$f(0)=\int_{-4}^{2(0)} g(t)\,dt$

$=\int_{-4}^{0} g(t)\,dt$ [area]

$-\frac{1}{2}(1)(1)+\frac{1}{2}(1)(1)+(1)(2)+\frac{1}{4}\pi(2)^2=\boxed{\pi+2}$

2. Find the value of $f(2)$.

$f(2)=\int_{-4}^{2(2)} g(t)\,dt$

$=\int_{-4}^{4} g(t)\,dt$

$=(\pi+2)(2)=\boxed{2\pi+4}$

3. Find the value of $f'(1)$.

$f'(x)=g(2x)\cdot 2$

$f'(1)=g(2\cdot 1)\cdot 2$

$f'(1)=g(2)\cdot 2$

$=(1)(2)=\boxed{2}$

4. Find the value of $f'(-2)$.

$f'(x)=g(2x)\cdot(2)$

$f'(-2)=g(2\cdot -2)\cdot(2)$

$g(-4)\cdot 2$

$-1\cdot 2=\boxed{-2}$

5. Find the value of $f''(2)$.

$f'(x)=g(2x)\cdot 2$

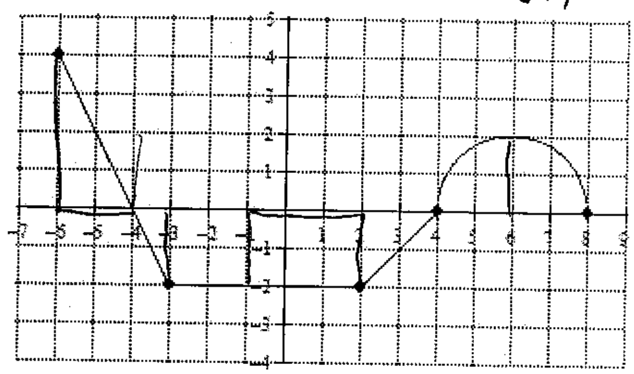
$f''(x)=2g'(2x)\cdot 2$

$f''(x)=4g'(2x)$

$f''(2)=4g'(2\cdot 2)=4g'(4)$

$f''(2)$ is undefined b/c $g(t)$ is not diff at $x=4$ since $g(t)$ is not cont at $x=4$

Given to the right is the graph of $f(t)$ which consists of three line segments and one semicircle. Additionally, let the function $g(x)$ be defined to be $g(x) = \int_{-1}^{x} f(t)dt$.



1. Find $g(-6)$.

$g(-6) = \int_{-1}^{-6} f(t)\,dt = -\int_{-6}^{-1} f(t)dt$

area

$\frac{1}{2}(2)(4) + -\frac{1}{2}(1)(2) - (2)(2)$

$4 - 1 - 4 = -(-1) = \boxed{1}$

2. Find $g(6)$.

$g(6) = \int_{-1}^{6} f(t)dt$

$-(2)(3) - \frac{1}{2}(2)(2) + \frac{1}{4}\pi(2)^2$

$-6 - 2 + \pi = \boxed{-8+\pi}$

3. Find $g'(6)$.

$g'(x) = f(x)(1)$

$g'(6) = f(6) = \boxed{2}$

4. Find $g'(2)$.

$g'(x) = f(x)(1)$

$g'(2) = f(2) = \boxed{-2}$

5. Find $g''(2)$. Give a reason for your your answer.

$g'(x) = f(x)$

$g''(x) = f'(x)$

$g''(2) = f'(2) =$ undefined b/c $f(t)$ has cusp at $x=2$ (not diff.)

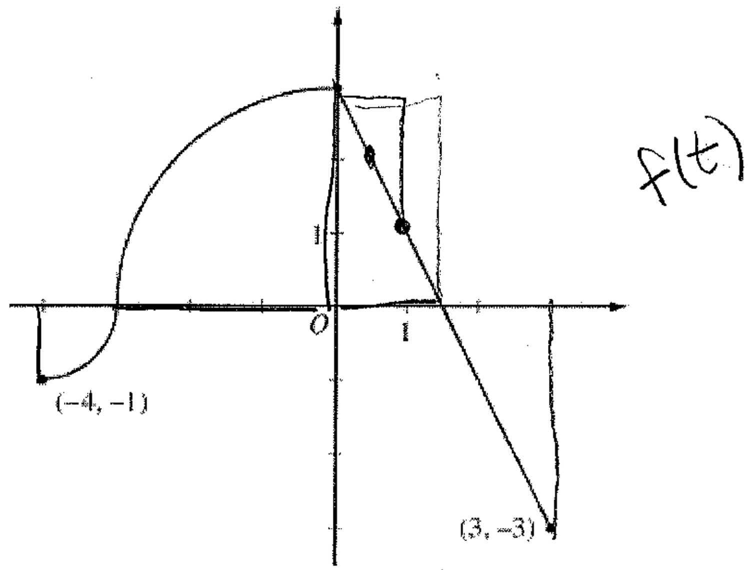
6. Find $g''(-4)$. Give a reason for answer.

$g''(x) = f'(x)$

$g''(-4) = \underbrace{f'(-4)}_{\text{slope of tangent line}} = \boxed{-2}$

AP Calculus AB
Unit 7 – Day 2 – Warm-up

Name: Answer Key*



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph consists of two quarter circles and one line segment, as show in the figure above. Let $g(x) = \frac{1}{2}x^2 + \int_0^x f(t)\,dt$.

Find the value of $g(3)$.

$$g(3) = \tfrac{1}{2}(3)^2 + \underbrace{\int_0^3 f(t)\,dt}_{\text{area}}$$

$$= 4.5 + \tfrac{1}{2}(1.5)(3) - \tfrac{1}{2}(1.5)(3)$$

$$\boxed{g(3) = 4.5}$$

Find the value of $g(-4)$.

$$g(-4) = \tfrac{1}{2}(-4)^2 + \int_0^{-4} f(t)\,dt$$

$$= 8 + -\underbrace{\int_{-4}^{0} f(t)\,dt}_{\text{area}}$$

$$= 8 + -\left[\tfrac{1}{4}\pi(3)^2 - \tfrac{1}{4}\pi(1)^2\right] = 8 - \tfrac{9}{4}\pi + \tfrac{1}{4}\pi$$

$$= \boxed{8 - 2\pi}$$

Find the value of $g'(3)$.

$$g'(x) = x + f(x)(1)$$

$$g'(3) = (3) + f(3)$$

$$= 3 + -3$$

$$= \boxed{0}$$

Find the value of $g''(2)$.

$$g'(x) = x + f(x)$$

$$g''(x) = 1 + f'(x)$$

$$g''(2) = 1 + \underbrace{f'(2)}_{\text{slope of tangent}}$$

$$= 1 + -2$$

$$= \boxed{-1}$$