AP Calculus
Unit 7 – Advanced Integration & Applications

Day 1 Notes: Second Fundamental Theorem of Calculus

Given the functions, $f(t)$, below, use $F(x) = \int_1^x f(t)dt$ to find $F(x)$ and $F'(x)$ in terms of x.

1. $f(t) = 4t - t^2$	2. $f(t) = \cos t$

Given the functions, $f(t)$, below, use $F(x) = \int_1^{x^2} f(t)dt$ to find $F(x)$ and $F'(x)$ in terms of x.

3. $f(t) = t^3$	4. $f(t) = 6\sqrt{t}$

Second Fundamental Theorem of Calculus

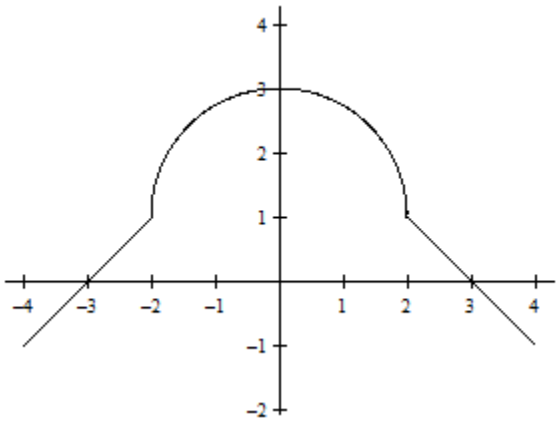
Complete the table below for each function.

Function	Find $F'(x)$ by applying the Second Fundamental Theorem of Calculus
$F(x)=\int_1^x \left(4t-t^2\right)dt$	$F(x)=\int_1^x \left(4t-t^2\right)dt$
$F(x)=\int_1^x (\cos t)dt$	$F(x)=\int_1^x (\cos t)dt$
$F(x)=\int_1^{x^2} t^3\,dt$	$F(x)=\int_1^{x^2} t^3\,dt$
$F(x)=\int_1^{x^2} 6\sqrt{t}\,dt$	$F(x)=\int_1^{x^2} 6\sqrt{t}\,dt$

Find the derivative of each of the following functions.

$F(x)=\int_{-2}^{2x}\sqrt{2-t^2}\,dt$	$G(x)=\int_{x^2}^{-3}e^{\cos t}\,dt$	$H(x)=\int_{0}^{\cos x}t^2\,dt$

Pictured to the right is the graph of $g(t)$ and the function $f(x)$ is defined to be $f(x)=\int_{-4}^{2x}g(t)dt$.



1. Find the value of $f(0)$.

2. Find the value of $f(2)$.

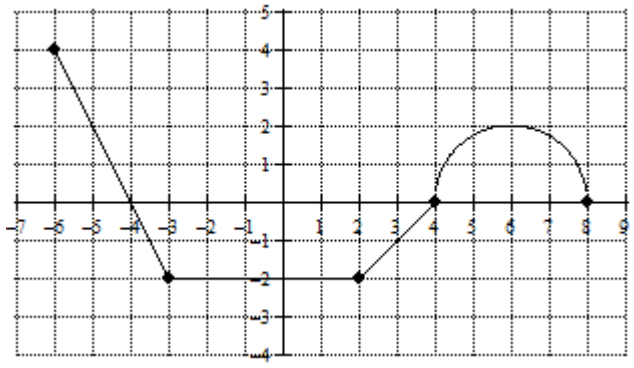
3. Find the value of $f'(1)$.

4. Find the value of $f'(-2)$.

5. Find the value of $f''(2)$.

Given to the right is the graph of $f(t)$ which consists of three line segments and one semicircle. Additionally, let the function $g(x)$ be defined to be $g(x) = \int_{-1}^{x} f(t)dt$.

1. Find $g(-6)$.



2. Find $g(6)$.

3. Find $g'(6)$.

4. Find $g'(2)$.

5. Find $g''(2)$. Give a reason for your your answer.

6. Find $g''(-4)$. Give a reason for answer.

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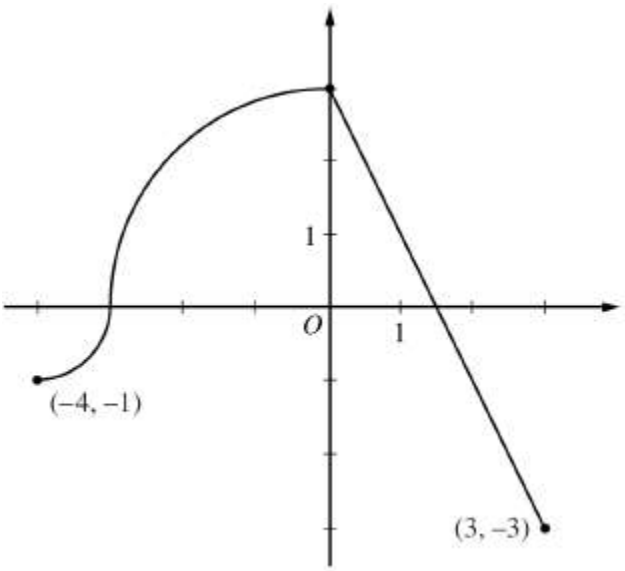
Unit 7 – Day 2 – Warm-up



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph consists of two quarter circles and one line segment, as show in the figure above. Let $g(x) = \frac{1}{2}x^2 + \int_0^x f(t)dt$.

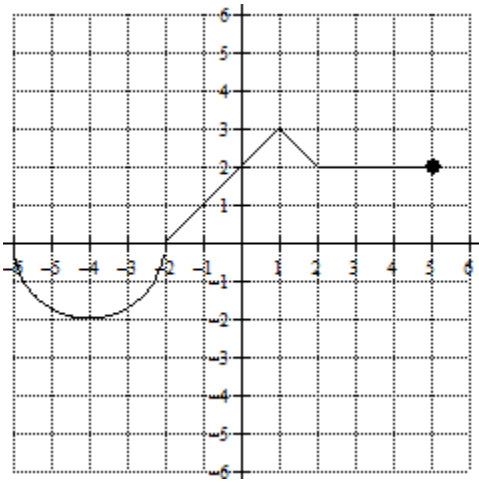
Find the value of $g(3)$.	Find the value of $g(-4)$.
Find the value of $g'(3)$.	Find the value of $g''(2)$.

AP Calculus AB **Name: _____**
Unit 7 – Day 1 – Assignment

Find the derivative of each of the following functions defined by integrals.

1. $g(x) = \int_2^{3x} (2t+3)dt$	2. $h(x) = \int_{-2}^{x^4} 3\sqrt{t}\, dt$
3. $f(x) = \int_{2x}^{-1} \left(t^2 + 2t\right)dt$	4. $H(x) = \int_{-5}^{\cos x} 2t^2\, dt$
5. $P(x) = \int_2^{x^2+2x} (3t-2)dt$	6. $f(x) = \int_{\ln x}^{2} \left(e^t + t\right)dt$

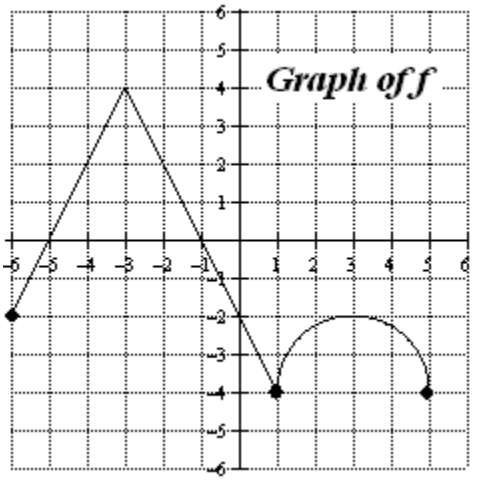
Pictured to the right is the graph of $f(t)$ and $F(x) = \int_{-6}^{2x} f(t)dt$. Use the graph and $F(x)$ to answer the questions 7 – 11.



7. Find the value of $F(0)$.	8. Find the value of $F\left(-\frac{1}{2}\right)$.	
9. Find the value of $F'(-2)$.	10. Find the value of $F'(2.5)$.	11. Find the value of $F''(0)$

Pictured to the right is the graph of f and $G(x) = \int_{-2}^{x} f(t)dt$. Use the graph to answer 12 – 15.

12. Find the value of $G(3)$.	13. Find the value of $G(-4)$.
14. Find the value of $G'(-2)$.	15. Find the value of $G''(-5)$.



If $g(x) = \int_0^x t^3 e^t \, dt$, find each of the following values in questions 16 – 17.

16. Find the value of $g'(1)$.	17. Find the value of $g''(1)$.

If $h(x) = \int_{x^2}^2 \sqrt{1+t^4} \, dt$, find each of the following values in questions 18 – 19.

18. Find $h'(x)$.	19. Find $h''(1)$.

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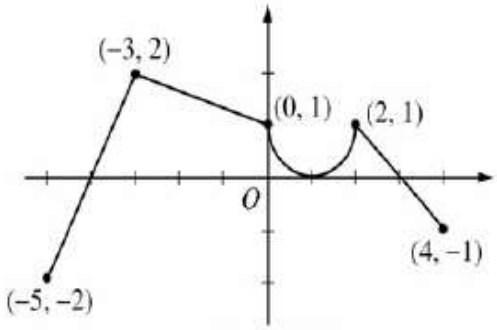
Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.