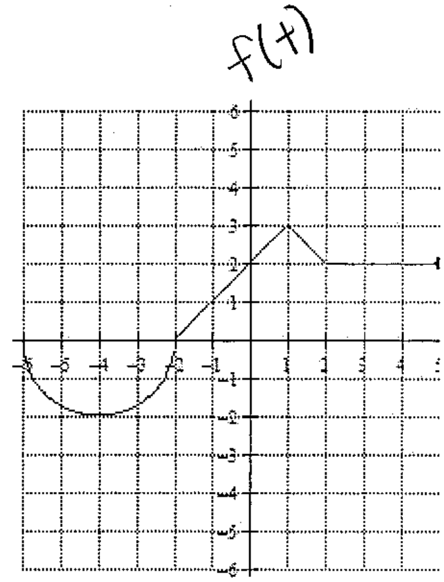
AP Calculus AB Name: Answer Key*
Unit 7 – Day 1 – Assignment

Find the derivative of each of the following functions defined by integrals.

1. $g(x)=\int_2^{3x}(2t+3)dt$ $g'(x)=(2(3x)+3)(3)$ $=(6x+3)(3)$ $=\boxed{18x+9}$	2. $h(x)=\int_{-2}^{x^4}3\sqrt{t}\,dt$ $h'(x)=(3\sqrt{x^4})(4x^3)$ $=(3x^2)(4x^3)$ $=\boxed{12x^5}$
3. $f(x)=\int_{2x}^{-1}(t^2+2t)dt$ $f(x)=-\int_{-1}^{2x}(t^2+2t)\,dt$ $f'(x)=-[(2x)^2+2(2x)](2)$ $=(-4x^2-4x)(2)$ $=\boxed{-8x^2-8x}$	4. $H(x)=\int_{-5}^{\cos x}2t^2\,dt$ $H'(x)=(2\cos^2 x)(-\sin x)$ $=\boxed{-2\cos^2 x\sin x}$
5. $P(x)=\int_2^{x^2+2x}(3t-2)dt$ $P'(x)=(3(x^2+2x)-2)(2x+2)$ $=(3x^2+6x-2)(2x+2)$ $=6x^3+6x^2+12x^2+12x-4x-4$ $=\boxed{6x^3+18x^2+8x-4}$	6. $f(x)=\int_{\ln x}^{2}(e^t+t)dt$ $f(x)=-\int_2^{\ln x}(e^t+t)\,dt$ $f'(x)=-[\cancel{e}^{\cancel{\ln x}}+\ln x]\left(\frac{1}{x}\right)$ $=\boxed{\dfrac{-x-\ln x}{x}}$

Pictured to the right is the graph of $f(t)$ and $F(x)=\int_{-6}^{2x} f(t)\,dt$. Use the graph and $F(x)$ to answer the questions 7 – 11.



7. Find the value of $F(0)$.

$F(0)=\int_{-6}^{2(0)} f(t)\,dt$

$=\int_{-6}^{0} f(t)\,dt$ area

$-\frac{1}{2}\pi(2)^2+\frac{1}{2}(2)(2)$

$\boxed{-2\pi+2}$

8. Find the value of $F\left(-\frac{1}{2}\right)$.

$F(-\tfrac{1}{2})=\int_{-6}^{2(-1/2)} f(t)\,dt$

$=\int_{-6}^{-1} f(t)\,dt$ area

$=-\frac{1}{2}\pi(2)^2+\frac{1}{2}(1)(1)$

$=\boxed{-2\pi+\frac{1}{2}}$

9. Find the value of $F'(-2)$.

$F'(x)=f(2x)(2)$

$F'(-2)=f(2\cdot-2)(2)$

$=f(-4)(2)$

$(-2)(2)$

$=\boxed{-4}$

10. Find the value of $F'(2.5)$.

$F'(x)=f(2x)(2)$

$F'(2.5)=f(2\cdot 2.5)(2)$

$=f(5)(2)$

$=(2)(2)$

$=\boxed{4}$

11. Find the value of $F''(0)$

$F''(x)=2f'(2x)(2)$

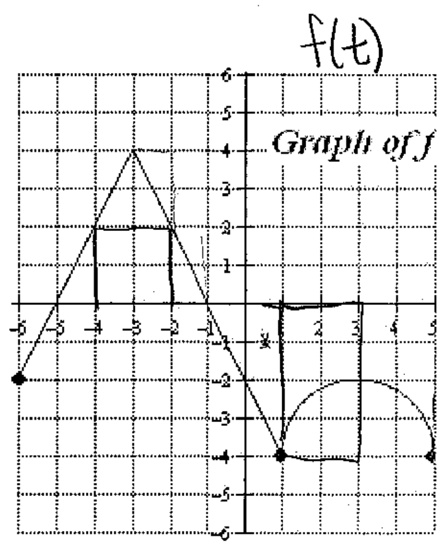
$=4f'(2x)$

$F''(0)=4f'(2\cdot 0)$

$=4f'(0) \leftarrow$ slope

$=4(1)=\boxed{4}$

Pictured to the right is the graph of f and $G(x)=\int_{-2}^{x} f(t)\,dt$. Use the graph to answer 12 – 15.



12. Find the value of $G(3)$.

$G(3)=\int_{-2}^{3} f(t)\,dt$ area

$=\frac{1}{2}(2)(1)-\frac{1}{2}(2)(4)$

$-\left[2(4)-\frac{1}{4}\pi(2)^2\right]$

$=1-4-8-\pi=\boxed{-11+\pi}$

13. Find the value of $G(-4)$.

$G(-4)=\int_{-2}^{-4} f(t)\,dt$

$=-\int_{-4}^{-2} f(t)\,dt$ area

$=-\left[(2)(2)+\frac{1}{2}(2)(2)\right]$

$=-4-2=\boxed{-6}$

14. Find the value of $G'(-2)$.

$G'(x)=f(x)(1)$

$G'(-2)=f(-2)$

$=\boxed{2}$

15. Find the value of $G''(-5)$.

$G'(x)=f(x)$

$G''(x)=f'(x)$

$G''(-5)=f'(-5)$ slope

$=\boxed{2}$

If $g(x) = \int_0^x t^3 e^t \, dt$, find each of the following values in questions 16 – 17.

16. Find the value of $g'(1)$.

$g'(x) = (x^3 e^x)(1)$

$g'(1) = (1)^3(e^1) = e$

17. Find the value of $g''(1)$.

$g'(x) = x^3 e^x$

$g''(x) = (3x^2)(e^x) + (x^3)(e^x)$

$g''(1) = (3(1)^2)(e^1) + (1^3)(e^1)$

$= 3e + e$

$= \boxed{4e}$

If $h(x) = \int_{x^2}^2 \sqrt{1+t^4} \, dt$, find each of the following values in questions 18 – 19.

18. Find $h'(x)$.

$h(x) = -\int_2^{x^2} \sqrt{1+t^4} \, dt$

$h'(x) = -\left[\sqrt{1+(x^2)^4} \cdot 2x\right]$

$= \boxed{-2x\sqrt{1+x^8}}$

19. Find $h''(1)$.

$h'(x) = -2x\sqrt{1+x^8}$

$h'(x) = -2x(1+x^8)^{\frac{1}{2}}$

$h''(x) = (-2)(1+x^8)^{1/2} + (-2x)\frac{1}{2}(1+x^8)^{-1/2}(8x^7)$

$= -2\sqrt{1+x^8} - \dfrac{8x^8}{\sqrt{1+x^8}}$

$h''(1) = -2\sqrt{1+1^8} - \dfrac{8(1)^8}{\sqrt{1+1^8}}$

$= -2\sqrt{2} - \dfrac{8}{\sqrt{2}}$

$= \dfrac{-2\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}} - \dfrac{8}{\sqrt{2}} = \dfrac{-4}{\sqrt{2}} - \dfrac{8}{\sqrt{2}}$

$= \boxed{\dfrac{-12}{\sqrt{2}}}$

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Question 5

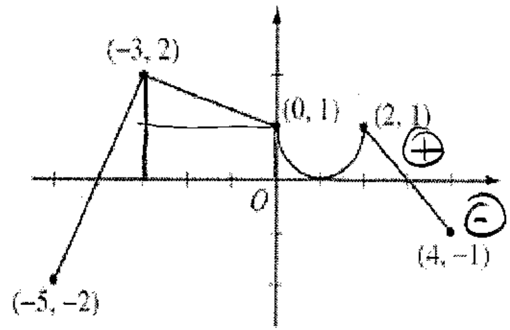
The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.



Graph of f

(a) $g(0) = \int_{-3}^{0} f(t)\,dt \overset{\text{area}}{=} \frac{1}{2}(1)(3) + (1)(3) = 1.5 + 3 = \boxed{4.5}$

$g'(x) = f(x) \cdot 1 \Rightarrow g'(0) = f(0) = \boxed{1}$

(b) $g(x)$ has a rel. max when $g'(x) = f(x)$ changes from positive to negative. $\rightarrow$ $\boxed{x=3}$

(c) Extreme Value Thm

possible abs min when $g'(x) = 0$ when $f(x) = 0$ & goes from neg → pos $x = -4$

$g(-5) = \int_{-3}^{-5} f(t)\,dt = -\int_{-5}^{-3} f(t)\,dt = \frac{1}{2}(1)(2) - \frac{1}{2}(1)(2) = 0$

$g(4) = \int_{-3}^{4} f(t)\,dt = 4.5 + [2(1) - \frac{1}{2}\pi(1)^2] + \cancel{\frac{1}{2}(1)(1)} - \cancel{\frac{1}{2}(1)(1)} = 4.5 + 2 - \frac{1}{2}\pi = 6.5 - \frac{1}{2}\pi$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\int_{-4}^{-3} f(t)\,dt = -[\frac{1}{2}(1)(2)] = -1$

$\boxed{\text{According to the EVT, the abs min of } g(x) = -1}$

(d) $g(x)$ has a point of inflection when $g''(x) = f'(x)$ changes signs. If $f'(x)$ changes signs then $f(x)$ has a rel. max or rel. min

$\boxed{x = -3,\ x = 1,\ x = 2}$