AP Calculus AB
Unit 6 – Review – Group Challenge

CALCULATOR ACTIVE:

1. If $\int_a^b f(x)dx = a+2b$, then what is the value of $\int_a^b [f(x)+5]dx$?

A. $a+2b+5$

B. $5b-5a$

C. $7b-4a$

D. $7b-5a$

$$\underbrace{\int_a^b f(x)\,dx}_{a+2b} + \underbrace{\int_a^b 5\,dx}_{5x\big]_a^b}$$

$$a+2b+[5(b)-5(a)] = -4a+7b$$

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

2. A table of values of a continuous function is shown above. If four equal subdivisions of the interval [0, 2] are used, what is the trapezoidal approximation of $\int_0^2 f(x)dx$?

A. 24

B. 12

C. 10

D. 16

$$\tfrac{1}{2}(.5)(3+3)+\tfrac{1}{2}(.5)(3+5)+\tfrac{1}{2}(.5)(5+8)+\tfrac{1}{2}(.5)(8+13)$$
$$1.5 + 2 + 3.25 + 5.25 = 12$$

3. A spherical tank contains 81.637 gallons of water at time $t=0$. For the next six minutes, water flows out of the tank at the rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of the six minutes?

A. 36.606

B. 45.031

C. 68.858

D. 126.668

start ← (0–6 min)

$$81.637 - \int_0^6 9\sin(\sqrt{t+1})\,dt$$

↑ math 9

81.637 − 45.031

36.606

4. The velocity of a particle, $v(t)$, is given by the function $v(t) = t^2 \cos(t+2)$. If the position of the particle at $t = 2$ is 10, what is the position when $t = 5$.

A. 16.778

B. 36.778

C. 48.789

D. −16.778

$\underbrace{\int_2^5 v(t)\,dt}_{\text{math 9}} = p(5) - p(2)$

$26.778 = p(5) - 10$

$p(5) = 36.778$

5. After being poured into a cup, coffee cools so that its temperature, $T(t)$, is represented by the function $T(t) = 70 + 110e^{-t/2}$, where t is measured in minutes and $T(t)$ is measured in degrees Fahrenheit. What is the average temperature of the coffee during the first four minutes after being poured?

A. 117.557 °F

B. 1356.996 °F

C. 470.226 °F

D. 5427.984 °F

$\frac{1}{4-0} \underbrace{\int_0^4 70 + 110e^{-t/2}}_{\text{math 9}}$

$\frac{1}{4}(470.226) = 117.557$

25 gallons to start

6. A half-full 50 gallon water tank begins leaking at the rate of $L(t) = 5e^{-\frac{(t-3)^2}{2}}$ gallons per minute, where t is measured in minutes. How much water remains in the tank after 5 minutes?

A. 37.769 gallons

B. 12.769 gallons

C. 1.353 gallons

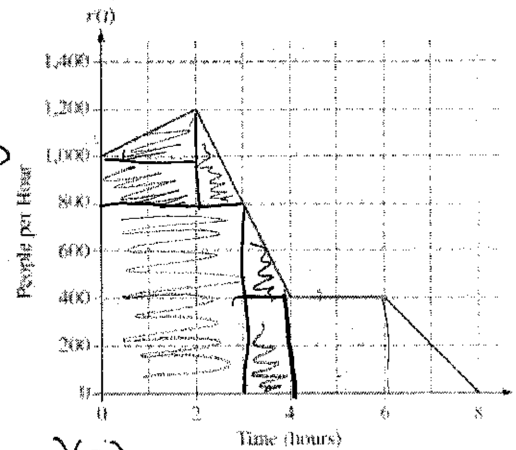
D. 12.231 gallons

$25 - \int_0^5 5e^{-\frac{(t-3)^2}{2}}\,dt$ math 9

$25 - 12.231 = 12.769$

rate

7. The graph to the right represents the rate at which people arrive at an amusement park ride throughout the day, where t is measured in hours from the time the ride begins operation. If there were 275 people in line when the ride began operation, How many people have waited in line for the ride after 4 hours?



A. 3800

B. 675

C. 400

D. 4075

start = 275 people

$275 + \underbrace{\int_0^4 r(t)}_{\text{area}}$

$(3)(800) + 2(200) + \frac{1}{2}(200)(2) + \frac{1}{2}(1)(400) + (1)(400) + \frac{1}{2}(1)(400)$

$275 + 2400 + 400 + 200 + 200 + 400 + 200 = 4075$

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8. Using a right Riemann sum over the given intervals, estimate $\int_5^{35} F(t)dt$.

t	5	13	22	27	35
$F(t)$	44	12	13	17	22

A. 730

B. 661

C. 474

D. 325

$(8)(12) + (9)(13) + (5)(17) + (8)(22)$

474

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9. At 10 a.m. the temperature at a ski resort begins to increase causing the snow to begin to melt at a rate defined by the equation $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. If there are 178 cubic yards of snow at that point, how much snow remains at 5 p.m. if no additional snow has been added and the temperature has continually increased throughout the day?

A. 90.646 cubic yards

B. 265.354 cubic yards

C. 87.025 cubic yards

D. 87.354 cubic yards

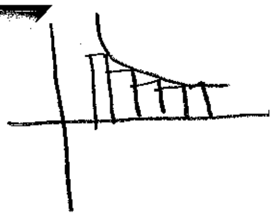
start → 178 yd^3 @ 10 am

5pm → 7 hrs

$178 - \int_0^7 10 + 8\cos\left(\frac{t}{3}\right)$ math9

$178 - 87.354$

90.646



above x-axis decr concave down

10. A function, $f(x)$ is such that $f(x) > 0$, $f'(x) < 0$ and $f''(x) < 0$ on the interval (2, 6). Which of the following statements can be made about the Riemann sum approximation on the interval?

Trapezoidal

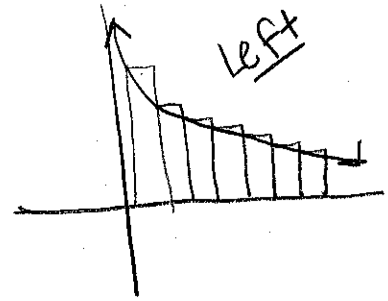
(A.) The left hand approximation will be an over approximation.

~~B.~~ The trapezoidal approximation will be an over approximation.

~~C.~~ The right hand approximation will be an over approximation.

~~D.~~ The left hand approximation will be an under approximation.

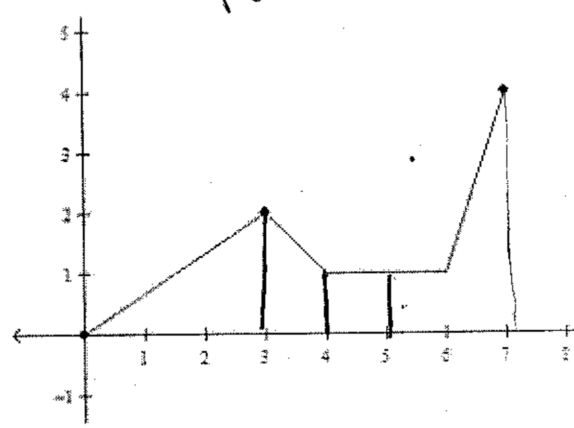
Left



NO CALCULATOR SECTION:

Pictured to the right is the graph of the function, $f(x)$.
Use the graph to answer questions 11 and 12.

f(x)



11. Find $\int_0^7 f'(x)dx$ $= f(7) - f(0)$

A. 5.5

B. 4

C. 9

D. undefined

D.N.E b/c cusp in graph so f'(x) not continuous

12. Find $\int_0^5 f(x)dx$. area

$\frac{1}{2}(3)(2) + \frac{1}{2}(1)(2+1) + (1)(1)$

$\frac{1}{2}(3)$

$3 + 1.5 + 1 = 5.5$

A. 5.5

B. 11

C. 13

D. 12.5

$\frac{2x^2}{2}-2x$ $\quad x^2-2x\Big]_k^2$

$(2)^2-2(2)-[k^2-2k]$ $\quad -k^2+2k=-3$

$4-4-k^2+2k$ $\quad -k^2+2k+3=0$

13. If $\int_k^2 (2x-2)dx = -3$, which of the following values is a possible value of k? $\quad k^2-2k-3=0$

$(k-3)(k+1)$

A. -2 B. 0 C. 1 D. -1

$3, -1$

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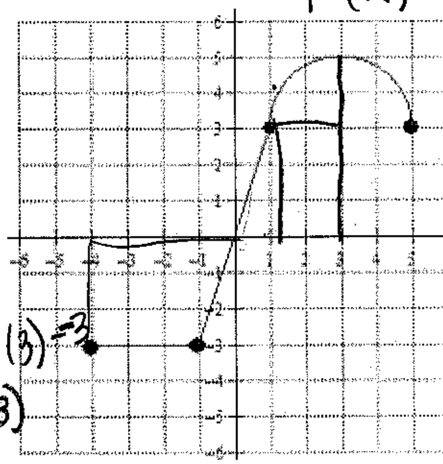
14. $\int \frac{x^4+2x^2}{x^3}dx =$ $\quad \int x+2x^{-1}$

A. $x+\frac{2}{x}+c$ B. $\frac{\frac{1}{5}x^5+x^2}{\frac{1}{4}x^4}+c$ $\quad \frac{x^2}{2}+\frac{2x^0}{0}$

C. $\frac{1}{2}x^2-\ln x+c$ D. $\frac{1}{2}x^2+2\ln x+c$ $\quad \frac{1}{2}x^2+2\ln x+C$

Pictured to the right is the graph of $f'(x)$, the derivative of a function $f(x)$. Use the graph for questions 15 and 16. $\quad f'(x)$



15. If $f(0) = -3$, what is the value of $f(3)$?

A. $10.5+\pi$

B. $10.5+2\pi$

C. $4.5+\pi$

D. $4.5+2\pi$

$\int_0^3 f'(x) = f(3)-f(0)$

$\frac{1}{4}(\pi)(2)^2+\frac{1}{2}(1)(3)+(2)(3)$

$\pi+1.5+6$ $\quad \pi+7.5=f(3)-3$

$\pi+4.5=f(3)$

16. Which of the following statements is/are true?

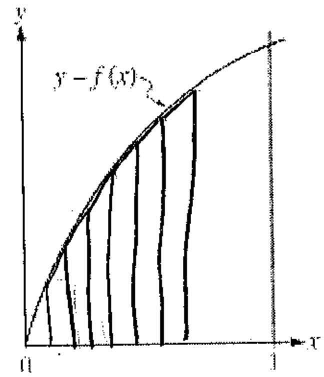
I. $f(x)$ is increasing on the interval $(0, 5)$. $f'(x)>0$

II. $f(x)$ has a point of inflection at $x = 3$.

III. $\int_{-4}^{0} f'(x)dx = 10.5$

A. I and II only B. II only C. II and III only D. I and III only

17. Which of the following sums give(s) an underestimate of the value of $\int_0^1 f(x)dx$ for the function to the right?



I. Left Sum II. Right Sum III. Trapezoidal Sum

A. I only

~~B~~. II only

C. I and III only

~~D~~. II and III only

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x	0	2	4	6
$f(x)$	4	k	8	12

18. The function f is continuous on the closed interval [0, 6] and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$, found with 3 subintervals of equal length is 52. What is the value of k?

A. 2

B. 6

C. 7

D. 10

$$\cancel{\tfrac{1}{2}}\cancel{(2)}\left[(4+k)+(k+8)+(8+12)\right] = 52$$

$$2k+32=52$$

$$2k=20$$

$$k=10$$

x	–3	–2	–1	0	1
$f(x)$	7	3	1	3	7
$f'(x)$	–5	–3	0	3	5
$f''(x)$	2	–1	–3	–2	0

19. Using the table of values above, find the value of $\int_{-3}^{1}[2f'(x)+3f''(x)]dx$.

A. 13

B. 30

C. 0

D. 9

$2\int_{-3}^{1} f'(x) + 3\int_{-3}^{1} f''(x)\,dx$

$2[f(1)-f(-3)] + 3[f'(1)-f'(-3)]$

$2\cancel{[7-7]} + 3[5-(-5)] = 3(10) = 30$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

20. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate of $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and a data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

A. 64.9

B. 68.2

C. 114.9

D. 116.6

$50 + \int_0^{15} R(t)$

$50 + [3(6.2) + 5(5.9) + 3(5.6)]$

$50 + [18.6 + 29.5 + 16.8]$

114.9