AP Calculus AB
Unit 6 – REVIEW

Name: Answer Key*

FREE RESPONSE PROBLEM #1
Calculator Permitted

Flows

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $\boxed{R(t) = 20\sin\left(\frac{t^2}{35}\right)}$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $\boxed{D(t) = -0.04t^3 + 0.4t^2 + 0.96t}$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

DRAIN

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

$$\int_0^8 R(t)\,dt = \int_0^8 20\sin\left(\frac{t^2}{35}\right)dt = \boxed{76.570 \text{ ft}^3}$$

math9

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

$$A(t) = 30 + \int_0^t R(t)\,dt - \int_0^t D(t)\,dt$$

$$A'(t) = R(t) - D(t)$$

$$A'(3) = 20\sin\left(\frac{32}{35}\right) - [-.04(3)^3 + .4(3)^2 + .96(3)]$$

$$= 5.086 - [5.4] = \boxed{-0.314 \text{ ft}^3/\text{hr}^2}$$

Since $A'(3)<0$, then the amount of water in the pipe at $t=3$ is decreasing

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

Extreme Value Theorem

$$A'(t)=0 \rightarrow R(t)-D(t)=0 \rightarrow \underbrace{R(t)}_{y_1} = \underbrace{D(t)}_{y_2} \rightarrow t = 3.272$$

Find intersection.

$$A(0) = 30 + \int_0^0 R(t)\,dt - \int_0^0 D(t)\,dt = 30$$

$$A(8) = 30 + \int_0^8 R(t)\,dt - \int_0^8 D(t)\,dt = 30 + 76.570 - 58.027 = 48.543$$

$$A(3.272) = 30 + \int_0^{3.272} R(t)\,dt - \int_0^{3.272} D(t)\,dt = 30 + 6.628 - 8.663 = 27.965$$

According to the E.V.T, the water in the pipe is at a minimum at $t = 3.272$ hrs.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

$$\boxed{48.543 + \int_8^t R(t)\,dt - \int_8^t D(t)\,dt = 50}$$

FREE RESPONSE PROBLEM #2
Calculator NOT Permitted

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

$$v'(16) = \frac{v(12)-v(20)}{12-20} = \frac{200-240}{12-20} = \frac{-40}{-8} = \boxed{5\ m/min^2}$$

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

$\int_0^{40} |v(t)| dt$ represents the total distance, in meters, that Joshua runs from $t=0$ to $t=40$ min.

$$\int_0^{40} |v(t)| dt = 12(200) + (8)(240) + (4)(\overset{\downarrow\ b/c\ |v(t)|}{+220}) + 16(150)$$

$$2400 + 1920 + 880 + 2400 = \boxed{7600\ m}$$

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

$$B'(t) = a(t) = 3t^2 - 12t$$

$$B'(5) = 3(5)^2 - 12(5) = 75 - 60 = \boxed{15\ m/min^2}$$

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

$$\frac{1}{10-0}\int_0^{10} t^3 - 6t^2 + 300\, dt = \frac{1}{10}\left[\frac{1}{4}t^4 - 2t^3 + 300t\Big|_0^{10}\right]$$

$$= \frac{1}{10}\left(\left[\frac{1}{4}(10)^4 - 2(10)^3 + 300(10)\right] - \left[\frac{1}{4}(0)^4 - 2(0)^3 + 300(0)\right]\right)$$

$$= \frac{1}{10}\left[2500 - 2000 + 3000 - 0\right] = \frac{1}{10}[3500] = \boxed{350\ m/min}$$

*

FREE RESPONSE PROBLEM #3
Calculator [redacted] Permitted

On a certain workday, the rate, measured in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$, the hours of operation. During the workday, the plant processes gravel at a rate of 100 tons per hour. When the unprocessed gravel arrives, it is emptied into 3 bins that will hold 200 tons of gravel each. If the three bins are full at any given time, delivery trucks must wait to unload. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel.

a. Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G'(5) =$ math8 $\frac{d}{dx}\left(90 + 45\cos\left(\frac{t^2}{18}\right)\right)_{x=5} = \boxed{-24.588 \text{ tons/hr}^2}$

Since $G'(5) < 0$, then the rate at which unprocessed gravel is arriving at the plant is decreasing at $t = 5$ hours.

b. Find $\int_0^8 G(t)\,dt$. Using correct units, interpret your answer in the context of the problem.

$\int_0^8 G(t)\,dt =$ math9 $\int_0^8 90 + 45\cos\left(\frac{t^2}{18}\right) = \boxed{825.551 \text{ tons}}$

$\int_0^8 G(t)dt$ represents the amount of change of unprocessed gravel that has arrived at the plant from $t=0$ to $t=8$ hrs.

c. Find an equation for $A(t)$, the amount of unprocessed gravel at the plant for any given time t. Using this equation, will the delivery truck arriving at $t = 7$ have to wait to unload its unprocessed gravel? Show your work and give a reason for your answer.

$\boxed{A(t) = 500 + \int_0^t G(t)\,dt - 100t}$

$A(7) = 500 + \int_0^7 G(t)\,dt - 100(7) = 500 + 779.257 - 700 = \boxed{579.257 \text{ tons}}$

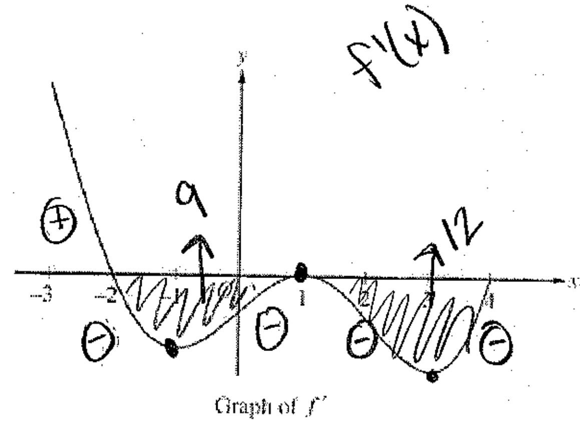
math9

At $t=7$, there are 579.257 tons of unprocessed gravel at the plant. The bins hold a total of 600 tons so the delivery truck does not have to wait to unload.

d. Write, but do not solve, an equation involving an integral that could be solved to determine the first time, t, when a truck would have to wait to unload its unprocessed gravel.

$$\boxed{500 + \int_0^t G(t)\,dt - 100t = 600}$$

FREE RESPONSE PROBLEM #4
Calculator NOT Permitted



The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

rel. max at $X=-2$ b/c
f'(x) changes from positive to negative.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

$f''(x)<0 \rightarrow$ $f'(x)$ is decR. $f'(x)<0$ (under x-axis)

$(-2,-1)\cup(1,3)$
b/c $f'(x)$ is decr & $f'(x)<0$

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

$f''(x)=0$ or when $f'(x)$ has rel. max or rel. min

$X=-1, \; X=1, \; x=3$

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

$\int_1^x f'(x) = f(x) - f(1)$

$f(x) = \int_1^x f'(x) + f(1)$

$f(4) = \underbrace{\int_1^4 f'(4)}_{\text{area}} + f(1)$

$= -12 + 3$

$f(4) = -9$

$f(-2) = \int_1^{-2} f'(x) + f(1)$

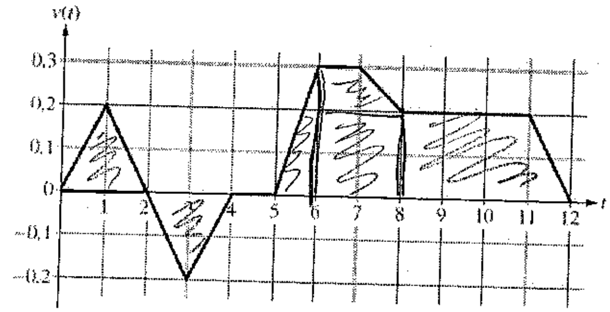
$= \underbrace{-\int_{-2}^1 f'(x)}\, + f(1)$

$= -(-9) + 3$

$f(-2) = 12$

FREE RESPONSE PROBLEM #5
Calculator Permitted

velocity



Caren rides her bicycle along a straight road from home to school, leaving home at $t = 0$ minutes and arrived at school at $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity, $v(t)$, in miles per minute is modeled by the piecewise-linear function whose graph is shown above.

a. Is Caren's speed increasing or decreasing at time $t = 7.5$ minutes? Completely explain your reasoning.

v(t) & a(t)

At $t = 7.5$ min, $v(t) > 0$ and $a(t) < 0$.
(above x-axis) ($v(t)$ is decr)

Since $v(t)$ & $a(t)$ have opposite signs, $\boxed{\text{speed is decreasing}}$.

b. Find the value of $\int_0^{12} |v(t)|\,dt$. Explain, using appropriate units, what this value represents in the context of the problem.

$$\int_0^{12} |v(t)|\,dt = \tfrac{1}{2}(2)(0.2) + \tfrac{1}{2}(2)(0.2) + \tfrac{1}{2}(1)(0.3) + \tfrac{1}{2}(0.1)(1+2) + (2)(0.2) + \tfrac{1}{2}(0.2)(3+4) = 1.8 \text{ miles}$$

$\int_0^{12} |v(t)|\,dt$ represents the total distance Caren rides from $t=0$ to $t=12$ min.

c. At some point in time, Caren realizes that she left her calculus homework at home and turned around to go back home to get it. At what time t does she turn around? Give a reason for your answer.

when $v(t)$ changes from positive to negative

$\boxed{t = 2 \text{ minutes}}$

nly do from t=5 b/c this is when she heads back to school from home after turning around

d. Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is measured in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

[Find distance]

aren: $\int_5^{12} v(t)\,dt = \tfrac{1}{2}(1)(0.3) + \tfrac{1}{2}(0.1)(1+2) + (2)(0.2) + \tfrac{1}{2}(0.2)(3+4) = 1.4 \text{ miles}$

arry: $\int_0^{12} w(t)\,dt = 1.6 \text{ miles}$ (use math 9)

| Caren lives closer to school than Larry. |

FREE RESPONSE PROBLEM #6
Calculator NOT Permitted

A particle is moving along a straight path. The velocity of the particle for $0 \le t \le 18$ is shown in the table below for selected values of t and velocity is a strictly increasing function.

t	0	3	6	9	12	15	18
$v(t)$ m/sec	0	7	10.5	12	13	14	14.5

rect

a. Using the midpoints of three subintervals of equal length, approximate the value of $\frac{1}{18}\int_0^{18} v(t)dt$. Using correct units, explain the meaning of the value of $\frac{1}{18}\int_0^{18} v(t)dt$.

$$\frac{1}{18}\int_0^{18} v(t)\,dt = \frac{1}{18}\left[6(7)+6(12)+6(14)\right] = \frac{1}{18}\left[42+72+84\right] = \frac{1}{18}(198)$$

Represents the $\boxed{\text{average velocity}}$ the particle travels from $t=0$ to $t=18$ seconds. $=\boxed{11 \text{ m/sec}}$

b. Find the average acceleration of the particle over the interval $6 \le t \le 18$. Express your answer using correct units.

$$\text{Avg Accleration} = \frac{v(6)-v(18)}{6-18} = \frac{10.5-14.5}{-12} = \frac{-4}{-12} = \boxed{\frac{1}{3}\text{ m/sec}^2}$$

c. Find an approximation of $v'(6)$. Using correct units, explain what this value represents and state, providing justification, if the speed of the particle is increasing or decreasing at $t = 6$?

$$v'(6) = \frac{v(3)-v(9)}{3-9} = \frac{7-12}{-6} = \frac{-5}{-6} = \boxed{\frac{5}{6}\text{ m/sec}^2}$$

$v(6) = 10.5$ & $a(6) = 5/6$, Since $v(6) > 0$ & $a(6) > 0$, then the $\boxed{\text{speed is increasing!}}$

d. Find $\int_3^9 v(t)dt$ using a left hand approximation of two subintervals of equal length. Then, use this value to find the position of the particle at $t = 3$ seconds if the position at $t = 9$ seconds is 60 meters, expression your answer using appropriate units.

$$\int_3^9 v(t)\,dt = 3(7)+(3)(10.5) = \boxed{52.5\text{ m}}$$

$$\int_3^9 v(t)\,dt = p(9)-p(3)$$

$$52.5\text{m} = 60 - p(3)$$

$$\boxed{p(3) = 7.5\text{ m}}$$